D-Brane and F-Theory Model Building

D 膜与 F 理论模型构建

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Abstract

摘要

We review recent progress in the construction of four-dimensional vacua of Type II string theory and F-theory which yield the Standard Model of particle physics (SM) or extensions thereof. In Type II orientifold compactifications, the SM gauge group and chiral spectrum arise from the open string sector of the theory, namely, from stacks of D-branes. The universal features of the chiral spectrum between various sets of D-branes allow for a general approach to build realistic models, which can be implemented in different setups. We describe the realization of this strategy in Type II Calabi-Yau orientifold compactifications and Rational Conformal Field Theories, discussing the specific model building rules and features of each setting. The same philosophy can be extended to F-theory constructions. These provide new model building possibilities, as they combine the localization properties of D-branes with exceptional gauge groups and their representations.

我们综述了构建 II 型弦理论和 F 理论四维真空的最新进展，这类真空可给出粒子物理的标准模型 (SM) 或其扩展。在 II 型定向偶紧化中，标准模型规范群和手征谱来自理论的开弦部分，即来自 D 膜叠。不同 D 膜集合之间手征谱的通用特征为构建现实模型提供了通用方法，该方法可在不同框架中实现。我们描述了该策略在 II 型卡拉比-丘定向偶紧化和有理共形场论中的实现，讨论了每种框架下特定的模型构建规则与特征。相同的理念也可推广到 F 理论构建中。F 理论结合了 D 膜的定域性质与例外规范群及其表示，提供了新的模型构建可能。

Keywords

关键词

String theory - D-branes - Orientifolds - F-theory - Conformal field theory - Model Building - String phenomenology

弦理论 - D 膜 - 定向对称面 - F 理论 - 共形场论 - 模型构建 - 弦唯象学

D-Branes and Orientifolds

D 膜与 O 平面

One of the aims of model building in string theory is to find string vacua whose spectrum of massless string excitations in four dimensions resembles as closely as possible the experimentally established and extremely successful Standard Model of particle physics (SM). Embedding the SM within string theory as a consistent theory of quantum gravity is more than merely a proof of principle; it can be viewed as a first step in a more ambitious program that hopes to understand some of the mysteries of particle physics from a string theoretic perspective.

弦论中模型构建的目标之一，是寻找这样的弦真空: 其四维下的无质量弦激发谱尽可能贴近经实验验证、极为成功的粒子物理标准模型 (SM)。将标准模型纳入作为自洽量子引力理论的弦论，远不止是原理性证明；这可以看作一个更宏大研究项目的第一步，该项目希望从弦论视角解答粒子物理的诸多谜题。

The SM has a gauge group (Throughout this article, we will not distinguish between the gauge algebra and gauge group unless stated explicitly. In particular, we will not discern between $O(N)$ and $SO(N)$ D-brane groups, as the difference cannot be determined by the perturbative arguments that we use.)

标准模型的规范群为 (全文除非另有说明，我们不区分规范代数与规范群。特别地，我们不对 $O(N)$ 和 $SO(N)$ D 膜群进行区分，因为我们使用的微扰论证无法得出二者的区别。)

$$SU(3) \times SU(2) \times U(1) \tag{1}$$

with matter in the representation

其物质场处于如下表示中

$$3 \times \left[\left(\mathbf{3}, \mathbf{2}, \frac{1}{6}\right) + \left(\overline{\mathbf{3}}, \mathbf{1}, -\frac{2}{3}\right) + \left(\overline{\mathbf{3}}, \mathbf{1}, \frac{1}{3}\right), + \left(\mathbf{1}, \mathbf{2}, -\frac{1}{2}\right) + (\mathbf{1}, \mathbf{1}, 1)\right]. \tag{2}$$

We will refer to these multiplets as Q, u^c, d^c, L , and e^+ , respectively. They are left-handed fermions. Right-handed fermions in the same representations do not exist, and for this reason the spectrum is called chiral. There may also exist non-chiral representations in nature. The most prominent candidates are singlets $(\mathbf{1}, \mathbf{1}, 0)$, which could play the role of right-handed neutrinos. They are in fact highly desirable in neutrino physics, but their existence is still not established. Such singlets are non-chiral, because their left- and right-handed components couple in the same way to the Standard Model gauge group. This means in particular that a mass term can be written down without breaking the Standard Model gauge group, i.e., without making use of the Standard Model Higgs mechanism. In general, many other non-chiral particles may exist. They can be in non-trivial Standard Model representations, and a Dirac or Majorana mass term for these particles is allowed by the Standard Model symmetries. We do not have any constraints on how large such a mass could be. However, a common assumption in nearly all attempts at string phenomenology is to allow such non-chiral particles to exist in the massless spectrum, and to implicitly assume that by some unspecified mechanism they acquire a mass, lower than the string scale but beyond the reach of current experiments. Hence a main goal of string phenomenology is to find string solutions that reproduce the spectrum (2) chirally. This is just a first step. If that is not possible, the whole idea is in serious doubt. A few string vacua have been identified in the literature where this spectrum is indeed realized exactly, but even then the string solution may differ from the SM in many more details, like for instance the strength of the couplings.

我们将这些多重态分别称为 Q, u^c, d^c, L 和 e^+ 。它们都是左手费米子，不存在相同表示下的右手费米子，因此该谱被称为手征谱。自然界也可能存在非手征表示，最典型的候选是单态 $(\mathbf{1}, \mathbf{1}, 0)$ ，它们可以充当右手中微子。实际上它们在中微子物理中是非常受欢迎的，但目前仍未证实它们存在。这类单态是非手征的，因为它们的左手和右手分量以相同方式与标准模型规范群耦合。这意味着我们可以直接写出质量项，而不需要破缺标准模型规范群，即不需要借助标准模型希格斯机制。一般来说，还可能存在许多其他非手征粒子，它们可以处于标准模型的非平凡表示中，标准模型对称性允许这些粒子拥有狄拉克或马约拉纳质量项。目前我们对这类质量的大小没有任何限制。然而，几乎所有弦唯象学研究都有一个共同假设: 允许这类非手征粒子存在于无质量谱中，并默认它们会通过某种未指明的机制获得质量，该质量低于弦标度，但超出当前实验的探测范围。因此，弦唯象学的一个主要目标就是找到能精确重现式 (2) 手征谱的弦解。这只是第一步。如果连这都无法实现，整个研究思路就会面临严重质疑。文献中已经找到了少数确实精确实现了该谱的弦真空，但即便如此，弦解仍可能在很多其他细节上与标准模型不同，例如耦合强度。

The vast majority of the literature is about supersymmetric realizations of the Standard Model, and this is also what we implicitly assume here, unless stated otherwise. Non-supersymmetric realizations exist, but in general they have serious stability issues. However, the main feature we focus on here, which is the gauge group and the chiral fermion spectrum, is anyway the same.

绝大多数文献都在讨论标准模型的超对称实现，除非另有说明，本文也默认采用这一假设。确实存在非超对称的实现，但一般来说它们存在严重的稳定性问题。不过本文关注的核心特征——规范群和手征费米子谱——在两种情况下 anyway 都是相同的。

Since 2011 we have known another particle in the ”light” (in Planckian units) spectrum: the Higgs boson.

In supersymmetric theories, the minimal way to accommodate it is to add two chiral supermultiplets to the massless spectrum, H_u and H_d . These two form a non-chiral pair, in the representation $\left(\mathbf{1},\mathbf{2},\frac{1}{2}\right)+\left(\mathbf{1},\mathbf{2},-\frac{1}{2}\right)$, and therefore they can develop a mass term. Hence they are part of the set of non-chiral particles which string theory should reproduce.

自 2011 年起，我们已经确认“轻”(普朗克单位下) 谱中还存在另一种粒子: 希格斯玻色子。在超对称理论中，容纳它的最小方式是在无质量谱中添加两个手征超多重态 H_u 和 H_d 。这两个手征超多重态构成一个非手征对，处于表示 $\left(\mathbf{1},\mathbf{2},\frac{1}{2}\right)+\left(\mathbf{1},\mathbf{2},-\frac{1}{2}\right)$ 中，因此它们可以形成质量项。因此它们也属于弦论应当重现的非手征粒子集合。

Historically, the goal of finding the chiral Standard Model spectrum within string theory was first achieved in the framework of compactifications of the ten-dimensional $E_8 \times E_8$ heterotic string, a theory of closed strings. Compactification to 4 dimensions reduces the gauge group rather naturally to E_6 with chiral matter in the 78-dimension representation, and from there one could follow the well-known GUT path down to the Standard Model, usually via the intermediate groups $SO(10)$ and $SU(5)$. This approach was first considered in 1984 [1]. Although the ”second string revolution” had been started in that same year by Green and Schwarz with a paper on open strings [2], that possibility was ignored for more than a decade. The ten-dimensional open string gauge group, $SO(32)$, looked far less promising with regard to the SM, and open strings added an extra complication that most - with the exception of a few courageous ones - preferred to avoid.

从历史来看，在弦理论中寻找手征标准模型谱的目标，最初是在十维 $E_8 \times E_8$ 杂化弦 (一种闭弦理论) 的紧致化框架下实现的。紧致化到四维后，规范群相当自然地约化为 E_6 ，得到 78 维表示中的手征物质，此后人们可以遵循熟知的大统一理论路径一路得到标准模型，通常会经过中间群 $SO(10)$ 和 $SU(5)$ 。该方法最早于 1984 年被提出 [1]。尽管“第二次弦革命”同年就由格林和施瓦茨通过一篇关于开弦的论文开启 [2]，但开弦的可能性被忽视了十多年。十维开弦的规范群 $SO(32)$ ，在得到标准模型方面看起来希望渺茫，而且开弦还带来了一个额外的难题——除了少数敢为人先的研究者外，大多数学者都倾向于避开这个问题。

D-branes and Chan-Paton Multiplicities

D 膜与陈-帕顿多重度

This all changed with the discovery of D-branes. At the endpoints of open strings, boundary conditions must be imposed on the two-dimensional worldsheet fields. It had been known for a long time that two kinds of boundary conditions were possible: Neumann and Dirichlet boundary conditions. If the former are imposed, the endpoints of the open string move through space-time at the speed of light. But if Dirichlet conditions are imposed, the endpoints of the string have a fixed space-time location. This implies the existence of a special point in space-time, which breaks translation invariance. While this option was immediately rejected by most people, it was realized around 1989 that one can impose Dirichlet boundary conditions in some directions of space-time, and Neumann conditions in others [3]. Then the open string endpoints are fixed in some directions, but can move freely in others. These endpoints then sweep out a plane, or a membrane, which was called a Dirichlet brane or D-brane for short. The existence of such a membrane does indeed break translation invariance in directions orthogonal to it, but not in directions parallel to it. It was then understood that our universe could live on top of such a brane, without any contradictions with translation invariance in our own space-time dimensions.

这一切都随着 D 膜的发现而改变。在开弦的端点，必须对二维世界面场施加边界条件。长久以来人们已经知道存在两种可能的边界条件: 诺依曼边界条件与狄利克雷边界条件。如果施加前者，开弦端点会以光速在时空中运动。但如果施加狄利克雷边界条件，弦端点在时空中的位置是固定的。这意味着时空中存在一个特殊点，会破坏平移不变性。虽然大多数人一开始就排除了这种可能性，但在 1989 年前后人们意识到，可以在部分时空方向施加狄利克雷边界条件，在其余方向施加诺依曼边界条件 [3]。此时开弦端点在部分方向固定，在其余方向可自由运动。这些端点会扫出一个平面，也即一张膜，被命名为狄利克雷膜，简称 D 膜。这种膜的存在确实会破坏垂直于膜方向的平移不变性，但不会破坏平行于膜方向的平移不变性。随后人们认识到，我们的宇宙可以存在于这样一张膜上，且不会与我们自身时空维度的平移不变性产生任何矛盾。

This suggests a picture where we are living on a four-dimensional space-time membrane, embedded in the ten-dimensional space-time of string theory. There would be six uncompactified directions. But this cannot work, because gravity still detects all of space-time, and would therefore not exhibit the $\frac{1}{r^2}$ behavior characteristic of Newtonian gravity. Hence the extra six dimensions must be compactified, but can remain relatively large as long as limits from fifth-force experiments are respected (Alternatively, space in the extra six dimensions may be warped, rather than flat.). All interactions besides gravity are restricted to the brane, and hence impose no constraints on the extra dimensions. From here more involved scenarios can be formulated, because one may consider several higher-dimensional branes on top of the four-dimensional space-time that also wrap cycles of the compact manifold, without constraining the strength of gravity. These D-branes can then intersect each other in the extra six dimensions, which is a mechanism to generate a 4d chiral spectrum [4]. This has led to the name ”intersecting D-brane models” [5-7]. See for instance [8-12] for reviews on the subject, to which we refer for the vast original literature, and Fig. 1 for a pictorial representation of the idea.

这就给出了一幅图景: 我们生活在一张四维时空膜上，它嵌入弦理论十维时空之中。原本会存在六个未紧致化的维度，但这并不成立，因为引力可以感知整个时空，因此不会表现出牛顿引力所特有的 $\frac{1}{r^2}$ 行为。因此额外的六个维度必须被紧致化，但只要满足第五力实验的限制，它们就可以保持相对较大的尺寸 (另外，额外六维的空间也可以是弯曲的，而非平坦的。)。除引力外的所有相互作用都被限制在膜上，因此不会对额外维度施加约束。在此基础上可以构建更复杂的场景: 我们可以考虑多张高维膜叠加在四维时空之上，它们同时缠绕在紧致流形的闭链上，且不会限制引力的强度。这些 D 膜可以在额外六维中彼此相交，这是产生四维手征谱的一种机制 [4]。这类模型因此得名“相交 D 膜模型”[5-7]。该方向的综述可参见例如 [8-12]，大量原始文献可在这些综述中找到，图 1 给出了这个想法的示意图。

Since the early days of string theory, it has been understood that one could consistently assign multiplicities to the boundaries of open strings. These are called Chan-Paton multiplicities [13]. The mode expansion of open strings always contains a massless vector boson, just as the mode expansion of closed strings always contains a massless rank-2 tensor field, the graviton. This massless vector boson behaves like a gauge boson. If there is a Chan-Paton multiplicity N , then there are in fact N^2 such gauge bosons, and by inspecting their interactions, one can verify that they gauge a group $U(N)$. There can be many distinct D-branes in a theory, each defining a place for open strings to end on. For simplicity, one may think of them as D-branes wrapping different cycles on a compactification manifold. Each such brane a can define a Chan-Paton multiplicity N_a . One may think of that multiplicity in terms of N_a D-branes stacked on top of each other, and filling our four-dimensional space-time. These are called space-time filling branes. In this situation, an observer in this space-time sees a gauge group:

早在弦理论发展初期，人们就已经认识到可以自洽地给开弦边界赋予多重度。这就是所谓的陈-帕顿多重度 [13]。开弦的模式展开总会包含一个无质量矢量玻色子，就像闭弦的模式展开总会包含无质量二阶张量场，即引力子一样。这个无质量矢量玻色子的行为与规范玻色子一致。如果陈-帕顿多重度为 N，那么实际上就会存在 N^2 个这样的规范玻色子，考察它们的相互作用可以验证，它们所对应的规范群是 $U(N)$。一个理论中可以存在多张不同的 D 膜，每张 D 膜都是开弦端点可以终止的位置。为简化起见，可以把它们看作是缠绕在紧致化流形不同闭链上的 D 膜。每个这样的膜 a 都可以定义一个陈-帕顿多重度 N_a。可以把这个多重度理解为 N_a 张 D 膜堆叠在一起，充满我们的四维时空。这类膜被称为时空填充膜。在这种情况下，该时空里的观测者会看到如下规范群:

$$U(N_a) \times U(N_b) \times U(N_c) \times \ldots \tag{3}$$

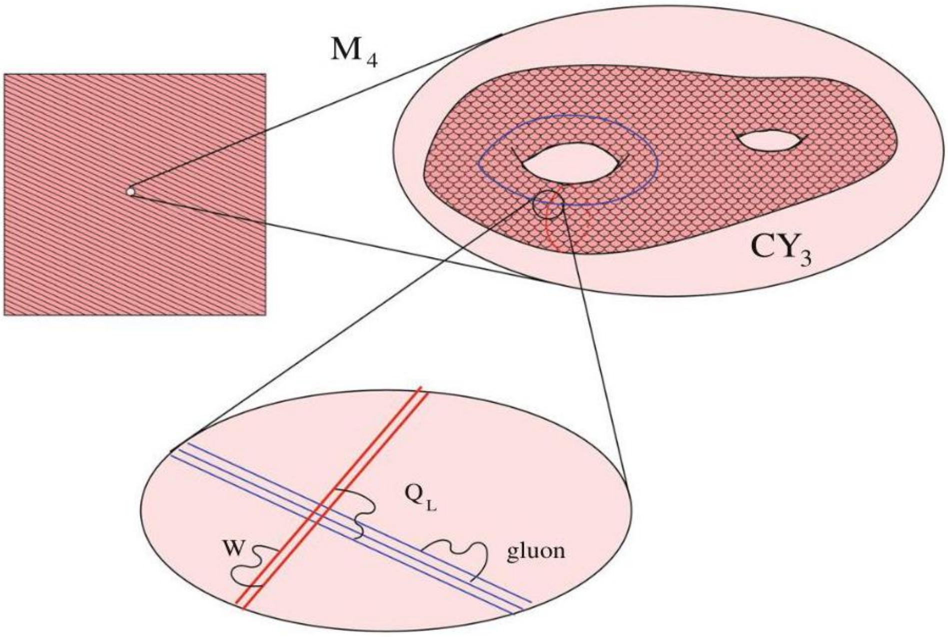


Fig. 1 Intersecting D-brane world scenario. (Figure taken from [10])

图 1 相交 D 膜世界图景。(图片引自 [10])

Oriented Strings: Groups and Representations

定向弦: 群与表示

Open strings with both ends on the same brane a give rise to a gauge group $U(N_a)$. The matter produced by such open strings includes vector bosons in the adjoint representation of $U(N_a)$. This immediately suggests the possibility of open strings having their endpoints on different branes, say a and b. It is clear that the physical particles produced by such strings must transform as a fundamental representation of $U(N_a)$ as well as that of $U(N_b)$. This is strongly suggested by the multiplicity N_aN_b of these states, and can be verified by working out the scattering amplitudes. Hence what one obtains from such strings are particles in the bi-fundamental representation $(\mathbf{N_a}, \mathbf{N_b})$. The mass and spin of these particles does not follow from this argument alone; we will return to this later.

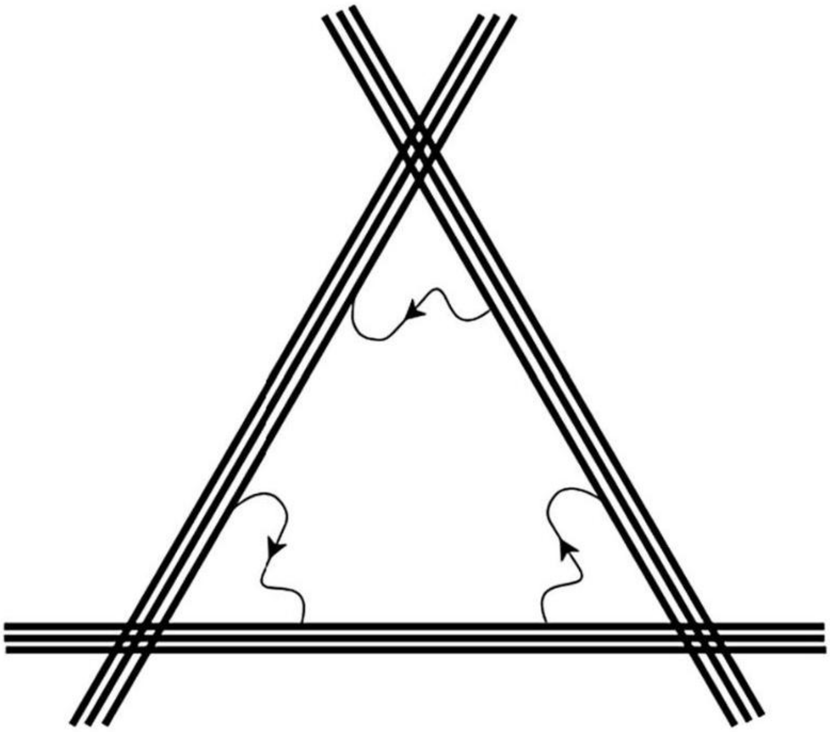
两端都位于同一膜 a 上的开弦会产生规范群 $U(N_a)$ 。这类开弦产生的物质包含属于 $U(N_a)$ 伴随表示的矢量玻色子。由此自然可以推想，开弦的端点也可以位于不同的膜上，比如 a 和 b 。显然，这类弦产生的物理粒子必须同时按照 $U(N_a)$ 的基础表示和 $U(N_b)$ 的基础表示变换，这一点可以由这些态的简并度 N_aN_b 推得，也可以通过计算散射振幅验证。因此，我们从这类弦得到的就是双基础表示 $(\mathbf{N_a}, \mathbf{N_b})$ 下的粒子。仅靠上述论证无法得出这些粒子的质量和自旋，我们稍后再回到这个问题。

The gauge group $U(N_a)$ has complex representations. Hence the multiplicity N_a can correspond to the representation $\mathbf{N_a}$ or its complex conjugate, $\overline{\mathbf{N_a}}$. What determines which one of the two we get? The open strings we are considering here are actually oriented. This defines a sense of direction along the open string, or in other words, one can consistently draw an arrow along it. Hence the two endpoints are distinct. Now we assign the endpoint with an outgoing arrow to $\mathbf{N_a}$ and the one with an incoming arrow to $\overline{\mathbf{N_a}}$. This is the origin of particles in the adjoint representation: open strings with both ends on the same brane produce a representation in the tensor product of $\mathbf{N_a}$ with $\overline{\mathbf{N_a}}$.

规范群 $U(N_a)$ 存在复表示，因此简并度 N_a 可以对应表示 $\mathbf{N_a}$ 或其复共轭表示 $\overline{\mathbf{N_a}}$ 。是什么决定了我们得到哪一种? 我们这里讨论的开弦实际上是定向的，这给开弦定义了方向，换句话说，我们可以一致地在弦上画出一个箭头，因此两个端点是可区分的。我们将出箭头指向的端点分配给 $\mathbf{N_a}$ ，入箭头指向的端点分配给 $\overline{\mathbf{N_a}}$ 。这就是伴随表示粒子的起源: 两端都位于同一膜上的开弦会在 $\mathbf{N_a}$ 和 $\overline{\mathbf{N_a}}$ 的张量积中产生一个表示。

Fig. 2 Trinification, an example of an oriented D-brane model

图 2 三重统一模型，定向 D 膜模型的一个实例



An Oriented String Model

定向弦模型

As a warm-up exercise, let us construct a simple brane configuration that will turn out to contain the Standard Model. Consider three stacks of $U(3)$ D-branes. Hence the gauge group is

作为热身练习，我们来构造一个简单的膜结构，最终它会包含标准模型。考虑三堆 $U(3)$ D 膜，因此规范群为

$$U(3) \times U(3) \times U(3) \ . \tag{4}$$

Now connect each pair of stacks with an oriented string, such that each stack contains one start- and one endpoint of the oriented strings, as shown in Fig. 2.

现在将每两堆膜用一根定向弦连接，使得每堆膜各有一根定向弦的起点和终点，如图 2 所示。

If these oriented strings have exactly three chiral (and hence massless) modes, the resulting spectrum is

如果这些定向弦恰好有三个手征 (即无质量) 模式，得到的能谱为

$$3 \times \left[\left(\mathbf{3}, \bar{\mathbf{3}}, \mathbf{1}\right) + \left(\mathbf{1}, \mathbf{3}, \bar{\mathbf{3}}\right) + \left(\bar{\mathbf{3}}, \mathbf{1}, \mathbf{3}\right) \right] .$$

This spectrum occurs naturally as a step in one of the symmetry-breaking paths from E_6 Grand Unification to the Standard Model. There are 27 massless states per family; 10 of them occur as mutually chiral pairs and there are 2 right-handed neutrinos. To arrive at the Standard Model, one has to break the last two factors to $SU(2) \times U(1)$ in a suitable way. This model has plenty of phenomenological issues, but there is a bigger problem we will have to deal with first.

这个能谱自然会出现在从 E_6 大统一到标准模型的其中一条对称性破缺路径中。每个家族有 27 个无质量态，其中 10 个以互手征对的形式存在，还有 2 个右手中微子。要得到标准模型，需要通过合适的方式将后两个因子破缺到 $SU(2) \times U(1)$ 。这个模型存在很多唯象学问题，但我们首先需要处理一个更大的问题。

From Oriented String Models to Orientifolds

从定向弦模型到定向轨形

The Need for O-Planes

O-平面的必要性

It turns out that in addition to branes, another ingredient is always needed, at least in supersymmetric theories: unoriented strings. These are strings, open or closed, without a definite orientation. If an oriented open string, with endpoints a and b , traces out a loop through space-time, the loop can only be closed by linking the a and b boundaries to themselves. The resulting string diagram is an annulus. But if a string is unoriented, the endpoints a and b are indistinguishable, so one can also link b to a when closing the loop. This results in a Moebius strip. Analogously, for orientable closed strings, the one-loop diagram is a torus, but in the case of unorientable closed strings, there is an additional diagram, the Klein bottle.

事实证明，至少在超对称弦理论中，除了膜之外始终还需要另一个组成要素: 未定向弦。这类弦无论是开弦还是闭弦，都没有确定的定向。对于带有端点 a 和 b 的定向开弦，它在时空中扫出一个回路时，只能通过将 a 边界和 b 边界各自连接来闭合回路，得到的弦图是一个环面。但如果弦是未定向的，端点 a 和 b 是不可区分的，因此闭合回路时也可以将 b 与 a 相连，得到的图形是莫比乌斯带。类似地，对于可定向闭弦，单圈图是亏格 1 环面，而对于不可定向闭弦，会额外出现一种图——克莱因瓶。

Constructions using unoriented strings are called ”orientifolds” [14, 15], by analogy to orbifolds. The idea is that one uses worldsheet parity as an orbifold symmetry. For instance, the Klein bottle amplitude can be thought of as a closed string sweeping out a closed loop, inverting its orientation before closing the loop; see [16] for further details. Unorientable strings enter the discussion not only as a logical possibility providing additional model building options but also because without them it is impossible to construct supersymmetric string vacua with spacetime filling D-branes.

沿用轨道流形的命名逻辑，使用未定向弦的构造被称为“定向模”[14,15]，其核心思想是将世界片奇偶性作为轨道对称使用。例如，克莱因瓶振幅可以理解为闭弦扫出闭合回路、并在闭合回路前反转自身定向；更多细节参见文献 [16]。不可定向弦进入我们的讨论不仅是因为它作为一种逻辑可能性提供了额外的模型构建选项，还因为如果没有它，就无法构造存在时空充满 D 膜的超对称弦真空。

To understand why, note that D-branes carry charge under the higher-form gauge potentials in the Ramond-Ramond (RR) sector of the superstring theory. Indeed, the worldvolume of a Dp -brane enjoys an electric coupling of the form $S = \mu_p \int_{\text{D}p\text{ -brane}} C_{p+1} + \ldots$ with μ_p the Dp -brane charge. In compactifications of string theory to four dimensions, a Dp-brane which extends along the visible 3+1 dimensions must fill a $(p-3)$ -dimensional closed subspace - a so-called $(p-3)$ - cycle $\Pi_{(p-3)}$ - on the six-dimensional compactification manifold X_6 . Due to their RR charge, the Dp -branes act as a source for the p -form gauge potential along the $9-p$ dimensions normal to the Dp -brane on the compact internal manifold. One can in fact characterize the charge under C_{p+1} in terms of the homology class $\left[\Pi_{(p-3)}\right] \in H_{(p-3)}(X_6, \mathbb{Z})$. This source is constrained by Gauss’law: the net charge on a compact space must vanish. In homological terms this amounts to requiring that a configuration of several D-branes satisfies:

要理解其中缘由，需要注意 D 膜在超弦理论的兰姆-兰姆 (RR) sector 中携带高阶规范势对应的电荷。确切来说，Dp 膜的世界体存在形式为 $S = \mu_p \int_{\text{D}p\text{ -brane}} C_{p+1} + \ldots$ 的电耦合，其中 μ_p 是 Dp 膜的电荷。在弦理论紧致化到四维的情形中，延伸在我们可见的 3+1 维时空的 Dp 膜，必须填满六维紧致化流形 X_6 上一个 $(p-3)$ 维闭子空间——即所谓的 $(p-3)$ -闭链 $\Pi_{(p-3)}$ 。由于自身的 RR 电荷，Dp 膜会成为紧致内流形上垂直于 Dp 膜的 $9-p$ 维方向上 p -形式规范势的源。实际上我们可以通过同调类 $\left[\Pi_{(p-3)}\right] \in H_{(p-3)}(X_6, \mathbb{Z})$ 来刻画 C_{p+1} 下的电荷。这个源受到高斯定律的约束: 紧致空间上的净电荷必须为零。用同调的语言来说，这要求多个 D 膜的构型满足:

$$\sum_a N_a \left[\Pi^a_{(p-3)}\right] = 0, \tag{5}$$

where we have included the multiplicity of D-branes wrapping each internal cycle. If a D-brane couples to other RR fields due to non-trivial worldvolume fluxes or curvature corrections, the induced D-brane charge must be cancelled as well, a condition that can be formulated in a similar fashion to (5).

其中我们计入了包裹每个内闭链的 D 膜的重数。如果 D 膜因非平凡世界体通量或曲率修正耦合到其他 RR 场，感应出的 D 膜电荷也必须被抵消，该条件可以用与 (5) 式类似的形式表述。

One can see that (5) cannot be satisfied in a supersymmetric D-brane configuration unless additional objects with opposite charge and tension to D-branes are introduced. The reason is that two mutually BPS D-branes will add both their charge (as a sum of homology classes) and their tension (as a sum of positive numbers). Therefore, the total tension in a supersymmetric D-brane configuration is always a linear function of the D-brane total charge, and a vanishing-charge condition like (5) cannot be compatible with only positive-tension objects.

不难看出，除非引入与 D 膜电荷相反、张力相反的额外物体，否则超对称 D 膜构型无法满足 (5) 式。原因在于，两个相互 BPS 的 D 膜的电荷 (同调类之和) 和张力 (正数之和) 都会相加。因此，超对称 D 膜构型的总张力始终是 D 膜总电荷的线性函数，零电荷条件 (5) 无法在仅存在正张力物体的情况下成立。

From a 4d viewpoint, the problem can be understood in terms of the cancellation of the $4d$ dilaton tadpole. The $4d$ dilaton has positive couplings (tension) to all D-branes, and hence the one-point functions with a single dilaton external lines is a sum of positive terms. Hence the 4d dilaton tadpole cannot cancel with only D-branes.

从四维视角来看，这个问题可以通过 $4d$ 型胀子蝌蚪图抵消来理解: $4d$ 型胀子与所有 D 膜的耦合都是正的 (即张力)，因此单外腿胀子的单点函数是若干正项的和，所以仅靠 D 膜无法抵消四维胀子蝌蚪图。

This requires the introduction of objects of negative charge and tension, the so-called orientifold p -planes, or Op -planes for short. As it turns out, such Op - planes are the fixed-point loci of involutions of the form $\Omega\mathcal{R}(-1)^N$, where Ω is the worldsheet parity operator that reverses the orientation of the string as described above and $\mathcal{R}$ acts as a geometric involution on the compactification space. Furthermore $(-1)^N$ is an operator that makes $\Omega\mathcal{R}(-1)^N$ square to the identity, and depends on the specific theory under consideration. In practice an Op -plane is specified by a submanifold or a sum of submanifolds $\Pi^{\mathrm{O}}_{(p-3)}$ of X_6 fixed by the geometric involution $\mathcal{R}$. Because this involution must be a symmetry of the compactification, if a D-brane internal worldvolume is not invariant under $\mathcal{R}$, there must be another D-brane located at $\Pi^{a'}_{(p-3)} = \mathcal{R}\Pi^{a}_{(p-3)}$, in order to identify their worldvolume theories. After Op -planes are introduced, the RR tadpole condition (5) is modified to

这就要求引入带负电荷与负张力的物体，即所谓的 orientifold p 平面，简称 Op 平面。研究表明，这类 Op 平面是形如 $\Omega\mathcal{R}(-1)^N$ 的对合的不动点轨迹，其中 Ω 是上文所述的反转弦取向的世界面奇偶算符，$\mathcal{R}$ 则对紧致化空间作用为几何对合。此外，$(-1)^N$ 是一个算符，它使得 $\Omega\mathcal{R}(-1)^N$ 平方等于单位算符，其具体形式取决于所研究的特定理论。实际中，一个 Op 平面由几何对合 $\mathcal{R}$ 固定的紧致化空间 X_6 的一个子流形或子流形之和 $\Pi^{\mathrm{O}}_{(p-3)}$ 来确定。由于该对合必须是紧致化的一个对称性，如果一个 D 膜的内部世界体在 $\mathcal{R}$ 作用下不不变，就必须在 $\Pi^{a'}_{(p-3)} = \mathcal{R}\Pi^{a}_{(p-3)}$ 处存在另一个 D 膜，以便对它们的世界体理论进行等同识别。引入 Op 平面后，RR 蝌蚪条件 (5) 被修改为

$$\sum_a N_a \left(\left[\Pi^{a}_{(p-3)}\right] + \left[\Pi^{a'}_{(p-3)}\right]\right) = Q_{\mathrm{O}p}\left[\Pi^{\mathrm{O}}_{(p-3)}\right], \tag{6}$$

where $Q_{\mathrm{O}p} = 2^{p-4}$ is minus the relative charge of an Op -plane and a Dp - brane wrapping the same $(p-3)$ -cycle. Now there is no obstruction to building a supersymmetric D-brane configuration. The simplest one is to place all D-branes wrapping homological cycles to the O-planes: $\left[\Pi^{a}_{(p-3)}\right] = \left[\Pi^{\mathrm{O}}_{(p-3)}\right]$ and $N_a = Q_{\mathrm{O}p}/2$.

where $Q_{\mathrm{O}p} = 2^{p-4}$ is minus the relative charge of an Op -plane and a Dp - brane wrapping the same $(p-3)$ -cycle. Now there is no obstruction to building a supersymmetric D-brane configuration. The simplest one is to place all D-branes wrapping homological cycles to the O-planes: $\left[\Pi^{a}_{(p-3)}\right] = \left[\Pi^{\mathrm{O}}_{(p-3)}\right]$ and $N_a = Q_{\mathrm{O}p}/2$.

To sum up, combining positive tension objects like D-branes in a supersym-metric fashion necessarily forces us to introduce negative-tension objects known as O-planes (Non-supersymmetric setups have aditional options, since one may introduce anti-branes.). These objects appear when one mods out an oriented configuration by an orientation-reversal symmetry of the theory. From the Type II perspective, they appear as non-dynamical objects of negative tension, whose microscopic nature can only be unveiled in a non-perturbative framework like F-theory.

总而言之，要以超对称的方式将正张力物体 (如 D 膜) 组合起来，我们就必须引入被称为 O 平面的负张力物体 (非超对称框架有额外选择，因为可以引入反 D 膜)。这些物体会在我们用理论的取向反转对称性对定向构型做商时出现。从 II 型弦的视角来看，它们是不具备动力学的负张力物体，只有在 F 理论这类非微扰框架中才能揭示其微观本质。

Unoriented Strings: Groups and Representations

未定向弦: 群与表示

One direct consequence of the presence of O-planes is that new groups and representations appear. Intuitively, if a D-brane is invariant under the orientifold projection, the D-brane group becomes real: instead of $U(N)$, it becomes either $SO(N)$ or $USp(N)$ (with N even). $U(N)$ groups are also still possible for some branes if their worldvolume is not fixed under the orientifold involution. One may think of this as two branes mapped to each other through the orientifold plane, a brane a and its orientifold image brane a' . For a more precise discussion of brane groups in orientifold models, see section "Gauge Groups" (In the RCFT literature, it is more common to refer to the orientifold image brane a' as the conjugate brane a^c . Both terms can be used interchangeably.).

O 平面对映的一个直接结果是出现了新的群和表示。直观来看，如果 D 膜在定向模投影下不变，D 膜的群就成为实群: 不再是 $U(N)$ ，它会变成 $SO(N)$ 或 $USp(N)$ (其中 N 为偶数)。如果部分膜的世界体积在定向模对合下不固定，$U(N)$ 群仍然可以存在。可以这么理解: 有两个膜通过定向平面互为镜像，一个膜是 a ，另一个是它的定向镜像膜 a' 。关于定向模模型中膜群的更详细讨论，参见“规范群”一节 (在 RCFT 文献中，更常将定向镜像膜 a' 称为共轭膜 a^c ，两个术语可以互换使用)。

All open string states are bi-fundamentals of one or two brane groups. Hence in D-brane model building, one must realize all Standard Model matter as bi-fundamental representations. In the unoriented case, open strings can connect to branes passing through an O-plane, and reversing their orientation. This implies that a bi-fundamental between two unitary branes can be of the form $\left(\mathbf{N},\overline{\mathbf{N}}\right)$ in addition to $(\mathbf{N},\mathbf{N})$. Here $\mathbf{N}$ denotes the fundamental (or vector) representation of $U(N)$, $SO(N)$ or $USp(N)$.

所有开弦态都是一个或两个膜群的双基本表示。因此在 D 膜模型构建中，所有标准模型物质都必须实现为双基本表示。在未定向的情形下, 开弦可以连接穿过 O 平面的膜, 并翻转自身定向。这意味着两个幺正膜之间的双基本表示除了 $(\mathbf{N},\mathbf{N})$ 之外, 还可以取 $\left(\mathbf{N},\overline{\mathbf{N}}\right)$ 的形式, 此处 $\mathbf{N}$ 表示 $U(N)$, $SO(N)$ 或 $USp(N)$ 的基本 (或矢量) 表示。

Furthermore, the two endpoints of an open string can be a fundamental representation on the same brane. This allows the existence of rank-2 tensors. These tensors can be symmetric and anti-symmetric, and will be denoted A or S, respectively. On unitary branes these tensors are complex representations of the brane group: one can have $\mathbf{A}, \overline{\mathbf{A}}, \mathbf{S}$, and $\overline{\mathbf{S}}$. Rank two tensors can also occur for real groups, and one can get adjoint representations of unitary groups if the open string endpoints are on a brane a and its orientifold image a' .

此外，开弦的两个端点可以对应同一膜上的基本表示，因此允许存在二阶张量。这些张量可以是对称张量或反对称张量，分别记为 S 和 A。在幺正膜上，这些张量是膜群的复表示: 可以存在 $\mathbf{A}, \overline{\mathbf{A}}, \mathbf{S}$ 和 $\overline{\mathbf{S}}$ 。实群也可以出现二阶张量；如果开弦端点分别位于膜 a 和它的定向镜像 a' ，还可以得到幺正群的伴随表示。

Only complex representations give rise to chiral matter. This means that if a theory contains left-handed fermions in the representation $n\,(\mathbf{N},\mathbf{N})+\bar{n}\left(\overline{\mathbf{N}},\overline{\mathbf{N}}\right)+\ m\left(\mathbf{N},\overline{\mathbf{N}}\right)+\bar{m}\left(\overline{\mathbf{N}},\mathbf{N}\right)$, this is equivalent to $(n-\bar{n})\,(\mathbf{N},\mathbf{N})+(m-\bar{m})\left(\mathbf{N},\overline{\mathbf{N}}\right)$, up to non-chiral matter. Rank-2 tensors of real groups and adjoints of unitary groups are chirally irrelevant, and although they may appear in the massless spectrum of specific D-brane configurations, these are states that a priori are not protected against becoming massive via a number of effects.

只有复表示能产生手征物质。这意味着如果一个理论包含 $n\,(\mathbf{N},\mathbf{N})+\bar{n}\left(\overline{\mathbf{N}},\overline{\mathbf{N}}\right)+\ m\left(\mathbf{N},\overline{\mathbf{N}}\right)+\bar{m}\left(\overline{\mathbf{N}},\mathbf{N}\right)$ 表示的左手征费米子，它就等价于 $(n-\bar{n})\,(\mathbf{N},\mathbf{N})+(m-\bar{m})\left(\mathbf{N},\overline{\mathbf{N}}\right)$ ，差别仅为非手征物质。实群的二阶张量和幺正群的伴随表示都不影响手征性，尽管它们可能出现在特定 D 膜构型的无质量谱中，但这些态先天上并不免受多种效应影响获得质量。

Despite this richer structure, it turns out that the 4d chiral spectrum obtained for D-branes in orientifold compactifications is quite universal. To describe it, one needs to define a chiral index I_{ab} between two D-branes, that is, a bilinear, antisymmetric tensor of their D-brane charges (not counting their multiplicity N_a). The expression for I_{ab} changes from one model building setup to another, but it is always of topological nature. If the D-brane with charge Π^b is not invariant under the orientifold action, there will be an orientifold image with charge $\Pi^{b'}$ and a corresponding index $I_{ab'}$. Finally, one can also extend this definition to include an index I_{aO} between a D-brane and the O-plane content of the compactification. Once that this index has been defined, the chiral spectrum arising from the open string sector of the compactification reads as in Table 1.

尽管结构更丰富，但研究发现，定向紧化中 D 膜得到的 4 维手征谱具有相当强的普适性。要描述该谱，我们需要定义两个 D 膜之间的手征指标 I_{ab} ，它是 D 膜荷的双线性反对称张量 (不包含其多重度 N_a)。I_{ab} 的表达式会因模型构建框架的不同而变化，但它始终具有拓扑性质。若荷为 Π^b 的 D 膜在定向模作用下不不变，就会存在一个荷为 $\Pi^{b'}$ 的定向模像，以及对应的指标 $I_{ab'}$ 。最后，我们还可以拓展该定义，纳入 D 膜与紧化中 O 面内容之间的指标 I_{aO} 。定义好该指标后，紧化开弦 sector 产生的手征谱如表 1 所示。

Table 1 Chiral spectrum of a D-brane orientifold compactification, in terms of the chiral index I_{ab} . For simplicity we are assuming that there are no D-branes with $SO\,(N)$ or $USp\,(N)$ gauge group. Π^a represents the charge of the D-brane. $\mathbf{S}_a, \mathbf{A}_a$ stand for the symmetric and anti-symmetric representations of $U\,(N_a)$

表 1 基于手征指标 I_{ab} 的 D 膜定向紧化手征谱。为简化起见, 我们假设不存在带有 $SO\,(N)$ 或 $USp\,(N)$ 规范群的 D 膜。Π^a 表示 D 膜的荷。$\mathbf{S}_a, \mathbf{A}_a$ 代表 $U\,(N_a)$ 的对称表示与反对称表示

Non-Abelian gauge group	$\prod_a SU(N_a)$
Massless $U(1)$ s Chiral fermions	$\sum_a c_a U(1)_a$ such that $\sum_a c_a \left([\Pi^a] - [\Pi^{a'}]\right) = 0$ $\sum_{a<b} I_{ab}\left(\mathbf{N}_a, \bar{\mathbf{N}}_b\right) + I_{ab'}\left(\mathbf{N}_a, \mathbf{N}_b\right)$ $\frac{1}{2}\left(I_{aa'} - I_{aO}\right)\mathbf{S}_a + \frac{1}{2}\left(I_{aa'} + I_{aO}\right)\mathbf{A}_a$

The fact that one has such a universal chiral spectrum for all perturbative orientifold models allows one to devise model building strategies that are independent of their specific realization, as will be discussed in the next section. Let us however stress that once one leaves the realm of perturbative constructions, new types of gauge groups, matter representations, and, consequently, model building possibilities arise. The reason is the appearance of non-perturbative bound states of strings which can have more than two endpoints, hence realizing, for instance, higher tensor representations or spinor representations. This will be described in detail in the context of F-theory in section ”F-Theory Model Building.”

所有微扰定向模模型都具备这类普适手征谱，这一点使得我们可以设计不依赖于具体实现的模型构建策略，我们将在下一节讨论这一点。但需要强调的是，一旦脱离微扰构造的范畴，就会出现新型规范群、物质表示，进而带来全新的模型构建可能性。原因在于，会出现端点多于两个的非微扰弦束缚态，例如可以实现高阶张量表示或旋量表示。我们将在“F 理论模型构建”一节中详细描述这一内容。

D-Brane Model Building: Generalities

D 膜模型构建: 概述

As it turns out, the Standard Model can be built very easily and naturally out of the limited set of bi-fundamentals and rank-2 tensors available already in perturbative D-brane models. In this section we outline the systematics underlying the search for Standard Model like vacua in Type II orientifolds. This approach can then be applied both in geometric Type IIA or Type IIB orientifolds (see section ”Type II Orientifolds”) and in conformal field theoretic models (section ”Rational Conformal Field Theories”). Model building in the non-perturbative generalization described by F-theory, in particular in the context of Grand Unified Theory (GUT) model building, is the topic of section ”F-Theory Model Building.”

事实证明，仅利用微扰 D 膜模型中已有的有限组双基本表示和二阶张量，就可以非常轻松自然地构建出标准模型。本节我们概述在 II 型定向扭面上寻找类标准模型真空的基础系统方法。该方法既适用于几何化的 IIA 型或 IIB 型定向扭面 (参见“II 型定向扭面”章节)，也适用于共形场论模型 (参见“有理共形场论”章节)。由 F 理论描述的非微扰推广框架下的模型构建，尤其是大统一理论 (GUT) 模型构建相关内容，是“F 理论模型构建”章节的主题。

Anomalies, Tadpoles, and Axions

反常、蝌蚪与轴子

In any chiral model, there is one important constraint to be taken into account: chiral anomaly cancellation. Anomalies cancel automatically in string theory, provided one satisfies all consistency conditions. The most important of these is in this context the cancellation of all RR tadpoles. Note that there may also be NS-NS tadpoles. They automatically cancel in supersymmetric models that are free from RR tadpoles. In non-supersymmetric setups, uncancelled NS-NS tadpoles imply instabilities, which is a serious problem, but not an inconsistency.

在任何手征模型中，都有一个重要约束需要考虑: 手征反常消除。只要满足所有自洽性条件，弦论中的反常就会自动抵消。在此框架下，这些条件中最重要的就是所有 RR 蝌蚪的消除。注意理论中也可能存在 NS-NS 蝌蚪。在不存在 RR 蝌蚪的超对称模型中，NS-NS 蝌蚪会自动消除。在非超对称框架中，未消除的 NS-NS 蝌蚪会引发不稳定性，这是一个严重问题，但不代表理论不自洽。

The first step toward building the Standard Model consists of assembling a set of branes whose spectrum of chiral fermions is the same as that of the SM. Unless one is extremely lucky, this set does not satisfy tadpole cancellation. In particular the NS-NS 4d dilaton tadpole can be oversaturated or undersaturated. In the former case, the total contribution of all branes plus the orientifold plane is positive. Then there is nothing one can do about this anymore. But if the total contribution is negative, one has the option of adding some additional branes to the configuration. This means that one chooses to assign a nonzero Chan-Paton multiplicity to some branes that are not part of the SM configuration. This must be done in such a way that no chiral particles are added to the spectrum: preferably no massless particles at all, or at least no chiral particles charged under the SM gauge group, dubbed chiral exotics. These additional branes are often referred to as a hidden sector. Such a sector may have several other uses, such as breaking supersymmetry and providing dark matter.

构建标准模型的第一步是组装一组 D 膜，使其手征费米子谱与标准模型一致。除非运气极好，否则这套 D 膜无法满足蝌蚪消除条件。特别是 NS-NS 四维胀子蝌蚪，其总贡献可能过高或过低。如果总贡献过高，所有膜加上定向反演平面的总贡献为正，此时就没有任何补救办法了。但如果总贡献为负，我们可以选择在构型中额外添加一些膜。这意味着我们为不属于标准模型构型的某些膜赋予非零的陈-帕顿多重度。添加过程必须保证不会往谱中引入手征粒子: 最好完全不引入无质量粒子，至少不能引入带有标准模型规范群荷的手征粒子，这类粒子被称为外来手征粒子。这些额外的膜通常被称为隐藏区。隐藏区还有其他多种用途，例如破缺超对称和提供暗物质。

Table 2 Properties of the fundamental (vector) **N** , symmetric **S** , and anti-symmetric **A** representations of $SU(N)$.$D(r)$ is the dimension of the representation $r, Q(r)$ is the $U(1)$ charge of r under the decomposition $U(N) = SU(N) \times U(1)$, and $I_2(r), I_3(r)$ are the quadratic and cubic anomaly coefficients (also known as Dynkin indices), respectively

表 2 **N** 基础 (矢量) 表示、**S** 对称表示和 **A** 反对称表示的性质: $SU(N)$.$D(r)$ 是表示的维数，$r, Q(r)$ 是 r 在分解 $U(N) = SU(N) \times U(1)$ 下的 $U(1)$ 荷，$I_2(r), I_3(r)$ 分别是二次和三次反常系数 (也称为 Dynkin 指标)

r	N	N	S	Š	A	A
$D(r)$	N	N	$\frac{N(N+1)}{2}$	$\frac{N(N+1)}{2}$	$\frac{N(N-1)}{2}$	$\frac{N(N-1)}{2}$
$Q(r)$	1	-1	2	-2	2	-2
$I_2(r)$	1	1	$N+2$	$N+2$	$N-2$	$N-2$
$I_3(r)$	1	-1	$N+4$	$-N-4$	$N-4$	$-N+4$

Non-abelian Anomalies

非阿贝尔反常

Since finding hidden sectors can be very laborious, it helps to eliminate some SM configurations at an early stage. This means first of all that all non-abelian anomalies must cancel. Since we are building the Standard Model, one may think that SM anomaly cancellation ensures this, but this is only partly true. Indeed, if one has realized the SM spectrum, anomaly cancellation ensures the absence of $SU(3)_{\mathrm{QCD}}$ anomalies.

由于寻找隐藏 sector 的过程往往十分繁琐，因此在早期阶段就排除部分标准模型构型很有帮助。这首先意味着所有非阿贝尔反常必须被抵消。我们在构建标准模型时，或许会认为标准模型自身的反常抵消已经足够保证这一点，但事实并非完全如此: 确实，当我们得到了标准模型能谱后，反常抵消可以保证不存在 $SU(3)_{\mathrm{QCD}}$ 反常。

In QFT, non-abelian anomaly cancellation is a condition on representations of $SU(N), N \geq 3$. In particular, different representations of $SU(N)$ contribute to the $SU(N)$ cubic anomaly as their cubic anomaly coefficient (see Table 2), and the sum of their contributions must vanish. In QFT this must be imposed by hand, or else gauge invariance must be dropped. In string theory all anomaly cancellations follow from some deeper consistency condition, such as modular invariance for closed strings. In QFT, the limit $N \geq 3$ arises because the group $SU(2)$ has pseudo-real representations, so that2is equivalent to $\overline{\mathbf{2}}$. Furthermore, $SU(1)$ is trivial. But in string theory $U(3), U(2)$, and $U(1)$ brane stacks are all on the same footing, and there is no reason to expect a lower limit on N.

在量子场论中，非阿贝尔反常抵消是对 $SU(N), N \geq 3$ 表示的约束条件。具体而言，$SU(N)$ 的不同表示对 $SU(N)$ 三次反常的贡献由各自的三次反常系数决定 (参见表 2)，所有贡献的总和必须为零。在量子场论中，这个条件必须被手动引入，否则就只能放弃规范不变性。而在弦论中，所有反常抵消都源自更深层的一致性条件，例如闭弦的模不变性。在量子场论中，极限 $N \geq 3$ 的出现是因为群 $SU(2)$ 拥有赝实表示，因此 2 等价于 $\overline{\mathbf{2}}$。此外，$SU(1)$ 是平凡的。但在弦论中 $U(3), U(2)$，且 $U(1)$ 膜堆地位完全平等，没有理由要求 N 存在下限。

In open strings the anomaly cancellation condition was first derived by Bianchi and Morales [17]. As expected, they found that non-abelian anomalies must cancel for $U(N)$ for all N, even if $N = 2$ or $N = 1$. If a candidate SM configuration contains $U(2)$ or $U(1)$ branes, the conditions must be checked, and if it not satisfied, the configuration can never be realized. We refer to this class of anomalies as "non-abelian" anomalies using QFT terminology, although $SU(2)$ is anomaly-free in QFT and $U(1)$ is abelian.

在开弦中，反常抵消条件最早由比安奇 (Bianchi) 和莫拉莱斯 (Morales) 推导得出 [17]。不出所料，他们发现对所有 N 的 $U(N)$ 而言，非阿贝尔反常都必须被抵消，即使 $N = 2$ 或 $N = 1$ 的情况也不例外。如果一个候选标准模型构型包含 $U(2)$ 或 $U(1)$ 膜，就必须检验该条件，若条件不满足，这个构型永远不可能实现。我们沿用量子场论的术语将这类反常称为“非阿贝尔”反常，尽管 $SU(2)$ 在量子场论中是无反常的，而 $U(1)$ 本身是阿贝尔的。

Let us see how the cubic non-abelian anomaly looks like in D-brane models. Using the spectrum of Table 1 and the anomaly coefficients of Table 2, we find:

我们来看 D 膜模型中三次非阿贝尔反常的具体形式。利用表 1 的能谱和表 2 的反常系数，我们得到:

$$\mathcal{A}_{SU(N_a)^3} = \sum_{r \text{ irrep}} I_{3,a}(r) = \sum_b N_b \left(I_{ab} + I_{ab'}\right) - 4I_{aO}, \tag{7}$$

which must vanish for any $SU(N), N \geq 3$, present in the model. In general one can see that the rhs vanishes when RR tadpole conditions are imposed, by using the appropriate generalization of (6) and bilinearity of the chiral index. As anticipated, this occurs even for $N_a < 3$. The reason is that otherwise some other anomalies would be left uncancelled, namely, mixed and abelian anomalies.

对于模型中存在的任意 $SU(N), N \geq 3$ ，该式必须等于零。一般来说，利用对 (6) 的合适推广以及手征指标的双线性性质，可以看出当施加 RR tadpole 条件时，右侧会等于零。正如之前预料的，即使对 $N_a < 3$ 这也成立。原因是若不满足这一点，就会残留其他未被抵消的反常，即混合反常和阿贝尔反常。

Mixed and Abelian Anomalies

混合反常与阿贝尔反常

Mixed and abelian anomalies are those that include $U(1)$ symmetries. Using again the content of Tables 1 and 2, they read:

混合反常与阿贝尔反常是包含 $U(1)$ 对称性的反常。同样利用表 1 和表 2 的内容，它们可表示为:

$$\mathcal{A}_{U(1)_a - SU(N_b)^2} = \sum_{r \text{ irrep}} Q_a(r) I_{2,b}(r) = \delta_{ab} \mathcal{A}_{SU(N_a)^3} + N_a \left(I_{ab} + I_{ab'}\right), \tag{8}$$

$$\mathcal{A}_{U(1)_a - U(1)_b^2} = \sum_{r \text{ irrep}} Q_a(r) Q_b(r)^2 = \delta_{ab} N_a \mathcal{A}_{SU(N_a)^3} + N_a N_b \left(I_{ab} + I_{ab'}\right),$$

(9)

where we assume that the abelian factors arise from $U(N_a) = SU(N_a) \times U(1)_a$ and r now runs over the irreps with respect to the groups involved in the anomaly. Here, abusing notation, we have denoted by $\mathcal{A}_{SU(N_a)^3}$ the rhs of (7), which is a well-defined quantity even for $N_a < 3$. It turns out that, in both cases, the term proportional to $I_{ab} + I_{ab'}$ is cancelled by a generalized Green-Schwarz mechanism. However, the first term has to cancel due to the D-brane configuration. That is why, in practice, one needs to impose the condition $\mathcal{A}_{SU(N)^3} = 0$ even for $N = 1, 2$.

其中我们假设阿贝尔因子来自 $U(N_a) = SU(N_a) \times U(1)_a$ ，r 现在遍历反常涉及群的不可约表示。此处我们简化记号，将 (7) 式的右侧记为 $\mathcal{A}_{SU(N_a)^3}$ ，即使对于 $N_a < 3$ 它也是定义良好的量。可以证明，两种情况下与 $I_{ab} + I_{ab'}$ 成正比的项都可通过广义格林-施瓦茨机制抵消。然而第一项必须由 D 膜构型本身抵消。因此实际操作中，即使对于 $N = 1, 2$ 也需要施加条件 $\mathcal{A}_{SU(N)^3} = 0$ 。

The generalized Green-Schwarz mechanism works by means of a mixing term of the longitudinal component of an abelian vector boson with an axion. If such term is present, the vector boson acquires a mass by absorbing the axion using the Stückelberg mechanism. This eliminates all vector bosons from the spectrum that couple to anomalous $U(1)$s , but the mechanism may also affect non-anomalous $U(1)$s . The result is that only the combinations of $U(1)$s that appear in Table 1 remain massless, and this may be bad or good: one the one hand it may make the Standard Model Y -boson - and hence the photon - massive, but on the other hand it may eliminate undesirable non-anomalous $U(1)$ s that occur in most models. Most frequently this occurs with vector bosons that involve $B-L$, a non-anomalous symmetry in the Standard Model when three right-handed neutrinos are added.

广义格林-施瓦茨机制通过阿贝尔矢量玻色子纵分量与轴子的混合项起作用。如果存在该混合项, 矢量玻色子会通过施蒂克尔贝格机制吸收轴子从而获得质量。这会将能谱中所有耦合到反常 $U(1)$s 的矢量玻色子移除，但该机制也可能影响非反常 $U(1)$s 。最终结果是只有表 1 中出现的 $U(1)$s 组合保持无质量，这有好有坏: 一方面它可能让标准模型 Y 玻色子 (也就是光子) 获得质量，另一方面它可以移除大多数模型中存在的多余非反常 $U(1)$ 。这种情况最常出现在包含 $B-L$ 的矢量玻色子上，当加入三个右手中微子后，$B-L$ 是标准模型中一种非反常对称性。

It is customary to distinguish local and global models. A local model has a Standard Model brane configuration and no non-abelian anomalies. Constructing it only requires knowledge of the chiral indices of the participating branes. If all tadpoles are cancelled and the photon remains massless, one speaks of a global model. Note that checking the latter feature cannot be done in a local model, as it requires knowledge of the full set of available axions, which cannot be derived from the brane configuration alone.

习惯上我们将模型区分为局域模型和全局模型。局域模型包含标准模型膜构型，且不存在非阿贝尔反常，构造它仅需要知道参与膜的手征指标。如果所有蝌蚪图都被抵消且光子保持无质量，则称之为全局模型。请注意，局域模型无法验证后者性质，因为它需要掌握全部可用轴子的信息，而这无法仅从膜构型得到。

The Simplest Examples

最简单的示例

In this section we will derive the simplest possible realizations of the Standard Model spectrum in terms of bi-fundamentals and rank-2 tensors. These are not explicit realisations, although we will indicate if such realizations are known. The steps toward an explicit realization are:

在本节中，我们将推导用双基本表示和二阶张量表示标准模型谱的最简单可行实现。这些并非显式实现，但我们会标注目前是否已知这类实现。得到显式实现的步骤如下:

1. Find a brane configuration that contains the Standard Model.

1. 找到一个包含标准模型的膜配置。

2. Check absence of cubic $U(N)$ anomalies, including $N=2$ and $N=1$.

2. 检查不存在三次 $U(N)$ 反常，包括 $N=2$ 和 $N=1$ 。

3. Find a realization of this configuration in terms of actual branes.

3. 用实际的膜找到该配置的一个实现。

4. Check absence of a mass for the Y -boson.

4. 检查确认 Y 玻色子没有质量。

5. Check tadpole cancellation, or cancel tadpoles by means of a hidden sector.

5. 检查蝌蚪图抵消，或通过隐藏 sector 实现蝌蚪图抵消。

Here we will limit ourselves to discuss steps 1 and 2. The broadest exploration of explicit realizations was done in the context of RCFT Gepner models in [18], where all five steps were considered. Models were collected if they passed step 4. Here we aim for maximal simplicity: the minimal number of branes, exact family repetition (so that we can focus on a single family), the minimal gauge group, and no superfluous non-chiral pairs. Since these guiding principles are debatable, we will make concessions if necessary.

本文我们仅讨论步骤 1 和步骤 2。对显式实现最全面的探究已在文献 [18] 的 RCFT 格普纳模型框架中完成，该工作考虑了全部五个步骤，通过了步骤 4 的模型被收录整理。本文我们以最大简化为目标: 最少的膜数量、精确的代重复 (因此我们可以专注于单个代)、最小规范群，且没有多余的非手征对。由于这些指导原则存在可争议性，必要时我们会做出让步。

Perhaps one's first thought about realizing the Standard Model with branes would be to take a $U(3)$ for QCD, a $U(2)$, or $USp(2)$ brane for the weak interactions and a $U(1)$ brane for the Y -charge (Note that $USp(2)$ has the same Lie algebra as $SU(2)$. The same is true for $O(3)$, but open strings would give matter in the three-dimensional vector representation of $O(3)$, which does not occur in the Standard Model.). However, associating the Y charge with a separate $U(1)$ brane cannot work, because the quark-doublet $\left(\mathbf{3},\mathbf{2},\frac{1}{6}\right)$ would then be a tri-fundamental, which do not occur in open string models.

用膜实现标准模型时，人们第一想法可能是给 QCD 用一个 $U(3)$ ，给弱相互作用用一个 $U(2)$ 或 $USp(2)$ 膜，给 Y 荷用一个 $U(1)$ 膜 (注意 $USp(2)$ 与 $SU(2)$ 有相同的李代数。$O(3)$ 也是如此，但开弦会给出 $O(3)$ 三维矢量表示的物质，这在标准模型中不存在)。然而，将 Y 荷关联到一个独立的 $U(1)$ 膜是行不通的，因为夸克二重态 $\left(\mathbf{3},\mathbf{2},\frac{1}{6}\right)$ 会因此成为三基本表示，而这在开弦模型中不存在。

The Y-Charge

超荷 Y

Hence the Y -charge must involve the $U(1)$ factor of $U(3)$, the $U(1)$ factor of $U(2)$ (unless $USp(2)$ is used), and perhaps one or more additional branes. We label the branes as a (for the QCD brane) b (for the weak brane), and $c, d, \ldots$ for any additional $U(1)$ branes. The Y -charge generator must then take the form:

因此 Y 荷必须包含 $U(3)$ 的 $U(1)$ 因子、$U(2)$ 的 $U(1)$ 因子 (除非使用 $USp(2)$)，还可能包含一个或多个额外膜。我们将膜标记为 a (QCD 膜)、b (弱相互作用膜)，$c, d, \ldots$ 对应任意额外的 $U(1)$ 膜。那么 Y 荷的生成元必然形式如下:

$$Y = \left(x - \frac{1}{3}\right) Q_a + \left(x - \frac{1}{2}\right) Q_b + \gamma Q_c + \delta Q_d + \ldots . \tag{10}$$

We follow here the conventions of [18], and assume that all branes are unitary and Q_i is the $U(1)$ generator of brane i . We normalize these generators so that a vector representation has charge $+1$. If the coefficient of any Q_i vanishes, one can replace the corresponding brane by a real one with a symplectic or orthogonal group.

我们此处遵循文献 [18] 的约定，假设所有膜都是幺正的，且 Q_i 是膜 i 的 $U(1)$ 生成元。我们对这些生成元归一化，使矢量表示的荷为 $+1$。若任意 Q_i 的系数为零，就可以将对应膜替换为携带辛群或正交群的实膜。

In the following we will examine all possibilities for assigning quarks and antiquarks, using the minimal number of branes, and for a single family.

下文我们将使用最少数量的膜，检验为单代夸克与反夸克分配荷的所有可能性。

The Quark Doublet

夸克二重态

The coefficients of Q_a and Q_b ensure that the quark doublet gets the correct Y -charge, assuming that the bi-fundamental we use for the quark doublet is $\left(\mathbf{N}, \overline{\mathbf{N}}, 0, \ldots\right)$. Alternatively one could use $(\mathbf{N}, \mathbf{N}, 0, \ldots)$. This is just a convention, but it is useful to choose all multiplets as if we are in an orientable string theory, i.e., a fundamental representation on one end and an anti-fundamental on the other, until we do not have a choice anymore. At that point we know that we have reached a nonorientable configuration. If $x = \frac{1}{2}$, then Q_b does not participate in Y , and we may use $USp(2)$ instead of $U(2)$ for brane b . This choice makes the configuration nonorientable. Note that in orientable configurations, the coefficients of Q_i are not fully determined by the SM spectrum: if we shift all coefficients by the same amount Δ , then Δ cancels out between the two open string ends.

Q_a 和 Q_b 的系数保证夸克二重态获得正确的 Y 荷，前提是我们用于夸克二重态的双基本表示为 $\left(\mathbf{N}, \overline{\mathbf{N}}, 0, \ldots\right)$。我们也可以选择使用 $(\mathbf{N}, \mathbf{N}, 0, \ldots)$。这只是一个约定，但将所有多重态按可定向弦论的情形来选取是更方便的，也就是开弦一端为基本表示，另一端为反基本表示，直到我们不得不做出改变为止。到那个时候我们就知道我们得到了一个不可定向构型。如果 $x = \frac{1}{2}$，那么 Q_b 不参与 Y，我们可以用 $USp(2)$ 代替 $U(2)$ 对应膜 b。这个选择会让构型成为不可定向的。注意在可定向构型中，Q_i 的系数并不由标准模型谱完全确定: 如果我们把所有系数都平移同一个量 Δ，那么 Δ 会在开弦的两端抵消。

Antiquarks

反夸克

Next we can try to assign the antiquarks u^c and d^c . The representation $\overline{\mathbf{3}}$ of the antiquark can be obtained in two ways: as a rank-2 anti-symmetric tensor, from an open string with both ends on the $U(3)$ brane, or as a $\overline{\mathbf{N}}$ endpoint of an open string. In the latter case, the other endpoint of that string has to end on another brane. One then has the four options summarized in Table 3.

接下来我们可以尝试分配反夸克 u^c 和 d^c 。反夸克的表示 $\overline{\mathbf{3}}$ 可以通过两种方式得到: 作为二阶反对称张量，来自两端都落在 $U(3)$ 膜上的开弦；或是作为开弦的 $\overline{\mathbf{N}}$ 端点。在后一种情况中，该开弦的另一端必须落在另一张膜上。共有四种选项，总结在表 3 中。

At this point all configurations, except (ii), are non-orientable. In the quark sector, all models in the literature necessarily have one of these for structures for a single family. With three families one has the option to make different choices per family, as long as the coefficients of Q_i match.

目前除 (ii) 外，所有构型都是不可定向的。在夸克领域，现有文献中的所有单代模型必然具有其中一种结构。如果是三代，只要 Q_i 的系数匹配，每一代都可以做不同选择。

The Lepton Doublet

轻子二重态

Now we can try to assign the lepton doublet $\left(\mathbf{1},\mathbf{2},-\frac{1}{2}\right)$. If we do not add more branes, there is a unique choice in all cases except (ii) : a bi-fundamental $(0,\mathbf{N},\mathbf{N})$ between branes b and c . In case (ii) there are two options: either $\left(0,\mathbf{N},\overline{\mathbf{N}},0\right)$ or $\left(0,\overline{\mathbf{N}},0,\mathbf{N}\right)$. In both cases the configuration remains orientable.

现在我们可以尝试确定轻子二重态 $\left(\mathbf{1},\mathbf{2},-\frac{1}{2}\right)$ 的位置。如果不添加额外膜，除 (ii) 外所有情况都只有唯一选择: 位于膜 b 和膜 c 之间的双基本表示 $(0,\mathbf{N},\mathbf{N})$ 。在 (ii) 的情况有两个选项: 要么是 $\left(0,\mathbf{N},\overline{\mathbf{N}},0\right)$ ，要么是 $\left(0,\overline{\mathbf{N}},0,\mathbf{N}\right)$ 。两种情况下该构型都保持可定向性。

Table 3 The four basic quark/antiquark configurations for one family. Columns 2-5 define the class. The last two columns specify the possible completions to a full family, if no further branes are added, and for generic x in case (iv)

表 3 一个代的四种基本夸克/反夸克构型。第 2 至 5 列定义了构型的类别。若不添加额外膜，且 (iv) 情况中存在一般 x 时，最后两列给出了完整代的可能补全方式

Class	u^c	d^c	x	y	δ	L	e^+
(i)	$\left(\overline{\mathbf{N}},0,\mathbf{N}\right)$	$\left(\overline{\mathbf{N}},0,\overline{\mathbf{N}}\right)$	$\frac{1}{2}$	$-\frac{1}{2}$	-	$(0,\mathbf{N},\mathbf{N})$ or $\left(0,\overline{\mathbf{N}},\mathbf{N}\right)$	$\left(0,0,\overline{\mathbf{S}}\right)$
(ii)	$\left(\overline{\mathbf{N}},0,\mathbf{N},0\right)$	$\left(\overline{\mathbf{N}},0,0,\mathbf{N}\right)$	x	x	$x-1$	$\left(0,\mathbf{N},\overline{\mathbf{N}},0\right)$ or $\left(0,\bar{\mathbf{N}},0,\mathbf{N}\right)$	$\left(0,0,\mathbf{N},\overline{\mathbf{N}}\right)$
(iii)	$(\mathbf{A},0,0)$	$\left(\overline{\mathbf{N}},0,\mathbf{N}\right)$	0	0	-	$(0,\mathbf{N},\mathbf{N})$	$\left(0,\overline{\mathbf{A}},0\right)$
(iv)	$\left(\bar{\mathbf{N}},0,\mathbf{N}\right)$	$(\mathbf{A},0,0)$	$\frac{1}{2}$	$-\frac{1}{2}$	-	$(0,\mathbf{N},\mathbf{N})$	$\left(0,0,\overline{\mathbf{S}}\right)$

The Left-Handed Positron

左手正电子

Finally we try to assign the charged lepton $(\mathbf{1},\mathbf{1},1)$. Being an $SU(2)$ singlet, it must come either from a string that does not end on brane b or an anti-symmetric tensor on brane b . In cases (i) and (iv) , this fixes it uniquely to $\left(0,0,\overline{\mathbf{S}}\right)$. In case (iii) the only option is $\left(0,\overline{\mathbf{A}},0\right)$. In case (ii) a natural choice is $\left(0,0,\mathbf{N},\overline{\mathbf{N}}\right)$. One could also use rank-2 tensors, but only for special values of x .

最后我们来确定带电轻子 $(\mathbf{1},\mathbf{1},1)$ 的归属。由于它是 $SU(2)$ 单态，它必然来自一根不终结于膜 b 的弦，或者来自膜 b 上的一个反对称张量。在情形 (i) 和 (iv) 中，这唯一将其确定为 $\left(0,0,\overline{\mathbf{S}}\right)$ 。在情形 (iii) 中，唯一的选择是 $\left(0,\overline{\mathbf{A}},0\right)$ 。在情形 (ii) 中，一个自然的选择是 $\left(0,0,\mathbf{N},\overline{\mathbf{N}}\right)$ 。我们也可以使用二阶张量，但仅当 x 取特殊值时才行。

Weak Interaction Anomalies

弱相互作用反常

The SM contains a quark doublet $\left(\mathbf{3},\mathbf{2},\frac{1}{6}\right)$ and a lepton doublet $\left(\mathbf{1},\mathbf{2},-\frac{1}{2}\right)$ per family. The gauge group $SU(2)$ is anomaly-free in quantum field theory. If it is realized as $USp(2)$ in string theory, this is also true. But if it is realized as $U(2)$ in string theory, there are anomalies not seen in field theory. We have to decide if the representation denoted "2" is actual2or $\overline{2}$. If we do that in a single family, it is immediately clear that there is no way to cancel the weak anomalies from the quark and the lepton doublets. There are four ways out of this:

标准模型每个代包含一个夸克双态 $\left(\mathbf{3},\mathbf{2},\frac{1}{6}\right)$ 和一个轻子双态 $\left(\mathbf{1},\mathbf{2},-\frac{1}{2}\right)$ 。规范群 $SU(2)$ 在量子场论中无反常。若它在弦论中实现为 $USp(2)$ ，该结论依然成立。但若它在弦论中实现为 $U(2)$ ，就会存在场论中不曾出现的反常。我们必须确定标记为“2”的表示是基本 2 还是 $\overline{2}$ 。如果我们仅考虑单个代，立刻就会发现无法抵消夸克和轻子双态带来的弱反常。共有四种解决方法:

1. If $x=\frac{1}{2}$, one may use $USp(2)$ instead of $U(2)$. Then there are no weak brane anomalies.

1. 如果 $x=\frac{1}{2}$ ，可以用 $USp(2)$ 代替 $U(2)$ 。此时不存在弱膜反常。

2. If $x=\frac{1}{2}$, one may also use $(\mathbf{N},\mathbf{N})$ as a quark doublet. Then one may drop exact family repetition, to write the three doublet as $2\left(\mathbf{N},\overline{\mathbf{N}}\right)+(\mathbf{N},\mathbf{N})$.

2. 如果 $x=\frac{1}{2}$ ，也可以用 $(\mathbf{N},\mathbf{N})$ 作为夸克双态。随后可以放弃精确的代复制，将三个双态写为 $2\left(\mathbf{N},\overline{\mathbf{N}}\right)+(\mathbf{N},\mathbf{N})$ 。

3. One may add a rank-2 anti-symmetric anti-tensor. This contributes +2 to the weak anomaly, cancelling the contribution -3 of the quark doublet, and +1 of the lepton doublet. This adds a singlet chiral particle with charge $1-2x$ to the spectrum. For the most common values of x , this is a left-handed positron or a neutrino.

3. 可以添加一个 2 阶反对称反张量。它对弱反常的贡献为 +2，可以抵消夸克双态的-3 贡献与轻子双态的 +1 贡献。这会在谱中添加一个带电荷 $1-2x$ 的单态手征粒子。对于 x 最常见的取值，这是一个左手正电子或中微子。

4. If there are two ways of realizing a lepton doublet with opposite weak anomalies, one may add a non-chiral pair. For example, suppose $(\mathbf{N}, X)$ and $\left(\overline{\mathbf{N}}, Y\right)$ are both lepton doublets, where X and Y are some combination of brane representations. Now one may use a combination $2\times(\mathbf{N}, X)+\left(\mathbf{N}, \bar{Y}\right)$ to get three times the contribution of a single lepton doublet, but a net number of only one chiral lepton doublet. This adds a non-chiral pair to the spectrum with the features of a Higgsino pair $H_u + H_d$.

4. 如果存在两种实现轻子二重态的方式，且二者弱反常相反，则可以添加一个非手征对。例如，假设 $(\mathbf{N}, X)$ 和 $\left(\overline{\mathbf{N}}, Y\right)$ 均为轻子二重态，其中 X 和 Y 是膜表示的某种组合。此时可以利用组合 $2\times(\mathbf{N}, X)+\left(\mathbf{N}, \bar{Y}\right)$ 得到单个轻子二重态贡献的三倍，但净手征轻子二重态数目仅为 1。这会在谱中添加一个非手征对，其性质符合希格斯诺对 $H_u + H_d$ 。

The Four Classes of Models

四类模型

Class (i)

(i) 类

This class can be made anomaly-free on the weak brane by means of mechanisms 1,2, and 3. The c brane anomaly cancellation is a bit awkward. The anomaly of the model as shown in the table would be -3 . To cancel it we may add an antisymmetric tensor $\mathbf{A}$ on brane c . This has anomaly -3 and ground state dimension 0, and hence there are no massless states at all in this sector. Although this may seem a bit weird, explicit realizations of such $U(1)$ branes were found in [18] in fully tadpole-free models.

该类可通过机制 1、2、3 在弱膜上实现无反常。c 膜的反常消除略显笨拙。如表中所示，该模型的反常为-3。为消除该反常，我们可以在膜 c 上添加一个反对称张量 $\mathbf{A}$ 。它的反常为-3，基态维度为 0，因此该扇区根本不存在无质量态。尽管这看起来有点奇怪，但文献 [18] 已经在完全无 tadpole 的模型中找到了这类 $U(1)$ 膜的显式实现。

But there is a more appealing way. We can add a fourth brane with the same contribution to Y as brane c , and realize the left-handed positron as $\left(0, 0, \overline{\mathbf{N}}, \overline{\mathbf{N}}\right)$.

但还存在一种更合理的方案: 我们可以添加一块第四膜，它对 Y 的贡献与膜 c 相同，并将左手正电子实现为 $\left(0, 0, \overline{\mathbf{N}}, \overline{\mathbf{N}}\right)$ 。

Then we may add a string $\left(0, 0, \mathbf{N}, \overline{\mathbf{N}}\right)$, which has vanishing Y charge, and connect the lepton doublet to brane d instead of brane c . Now all anomalies on branes c and d cancel. Furthermore, if we use mechanism

1 or 2 on the weak brane, we get a model entirely built out of bi-fundamentals. If we take the weak group as $USp(2)$ and assume mechanism 1, the full chiral spectrum is

随后我们可以添加一根弦 $\left(0,0,\mathbf{N},\overline{\mathbf{N}}\right)$，它的 Y 荷为零，将轻子二重态连接到膜 d 而非膜 c。现在膜 c 和膜 d 上的所有反常都被消除。此外，如果我们在弱膜上使用机制 1 或 2，就能得到一个完全由双基本表示构建的模型。如果我们取弱群为 $USp(2)$ 并假设采用机制 1，完整手征谱为

$$3\times(\mathbf{N},\mathbf{N},0,0)\,Q$$

$$3\times\left(\overline{\mathbf{N}},0,\mathbf{N},0\right)\ u^c$$

$$3\times\left(\overline{\mathbf{N}},0,\overline{\mathbf{N}},0\right)\ d^c$$

$$3\times(0,\mathbf{N},0,\mathbf{N})\ L$$

$$3\times\left(0,0,\overline{\mathbf{N}},\overline{\mathbf{N}}\right)\ e^+$$

$$3\times\left(0,0,\mathbf{N},\overline{\mathbf{N}}\right)\ n$$

This spectrum is shown in Fig. 4. It is the most-studied class of brane models, first explored in detail in [19]. Numerous examples and variations have been found in subsequent papers.

该谱如图 4 所示。这是被研究最多的一类膜模型，最早由文献 [19] 进行详细探究。后续论文中已经找到了大量实例与变体。

A noteworthy feature is the presence of a baryon and a lepton brane: all quarks attach to brane a and all leptons to brane d . In fact, branes a and d have the same intersections with branes b and c . One may combine branes a and d into a $U(4)$ stack, and extend the $U(1)$ group on brane 2 to $U(2)$ or $USp(2)$ to get a left-right symmetric model. Combining all this one obtains a $SU(4)\times SU(2)_L\times SU(2)_R$ Pati-Salam model; see [20,21] and section "Type II Orientifolds" for specific realizations of this idea.

一个值得注意的特点是存在重子膜和轻子膜: 所有夸克都连接到膜 a , 所有轻子都连接到膜 d 。实际上, 膜 a 和膜 d 与膜 b 、膜 c 的交点完全相同。可以将膜 a 和膜 d 合并为一个 $U(4)$ 层, 将膜 2 上的 $U(1)$ 群扩展为 $U(2)$ 或 $USp(2)$, 得到左右对称模型。组合所有这些结构可以得到 $SU(4)\times SU(2)_L\times SU(2)_R$ 帕蒂-萨拉姆模型；关于该思路的具体实现参见文献 [20,21] 和“II 型定向轨形”章节。

The unitary factors of branes a and d are anomalous and therefore broken by axion mixing. They remain as global baryon number (B) and lepton number (L) symmetries. The linear combination $B-L$ is anomaly-free, and the corresponding gauge boson may or may not become massive. This can only be decided by examining the full global model, not just the local configuration. In [18] global examples were found where the $B-L$ photon acquires a mass. Furthermore, examples were found with tadpole cancellation without any additional branes, both with a massless and a massive $B-L$ photon.

膜 a 和膜 d 的幺正因子是反常的，因此会被轴子混合破缺。破缺后它们仍然作为整体重子数 (B) 和轻子数 (L) 对称性存在。其线性组合 $B-L$ 是无反常的，对应的规范玻色子可能获得质量也可能不获得质量。这只能通过检验完整整体模型确定，仅靠局部构型无法判断。文献 [18] 中找到了整体实例，其中 $B-L$ 光子获得了质量。此外还找到了不添加任何额外膜就能实现 tadpole 抵消的例子，同时包含无质量和有质量的 $B-L$ 光子两种情况。

Bi-fundamentals between branes b and c are not used to build the fermion spectrum. But these states have precisely the right quantum numbers to be identified as supersymmetric Higgs multiplets H_d and H_u . They have zero lepton number, so that potentially dangerous perturbative couplings such as LH_u are automatically forbidden.

膜 b 和膜 c 之间的双基本表示并不用来构建费米子谱。但这些态恰好拥有正确的量子数，可以被确定为超对称希格斯多重态 H_d 和 H_u 。它们的轻子数为零，因此 LH_u 这类存在潜在危险的微扰耦合会被自动禁戒。

Class (ii)

(ii) 类

This class is characterized by having separate branes for the u^c and d^c endpoints. Models of this kind were first explored in [22, 23]. However, weak anomaly cancellation was not considered in these papers. For arbitrary values of x , only mechanism 4 is available. Using that mechanism one arrives at a fully orientable brane configuration:

该类模型的特征是 u^c 和 d^c 端点分别对应不同的膜。这类模型最早在文献 [22, 23] 中被研究，但这些论文没有考虑弱反常抵消。对于任意取值的 x ，仅机制 4 适用。利用该机制可得到完全可定向的膜构型:

$$3\times\left(\mathbf{N},\overline{\mathbf{N}},0,0\right)\ Q$$

$$3\times\left(\overline{\mathbf{N}},0,\mathbf{N},0\right)\ d^c$$

$$3\times\left(\overline{\mathbf{N}},0,0,\mathbf{N}\right)\ u^v$$

$$6\times\left(0,\mathbf{N},\overline{\mathbf{N}},0\right)\ L$$

$$3\times\left(0,\mathbf{N},0,\overline{\mathbf{N}}\right)\ L^c$$

$$3\times\left(0,0,\mathbf{N},\overline{\mathbf{N}}\right)\ e^+$$

There is a superfluous non-chiral pair $L+L^c$ per family, which has the quantum numbers of a Higgs pair, but is not distinguished from the lepton doublet by its quantum numbers. There is no left-handed antineutrino.

每个家族存在一个多余的非手征对 $L+L^c$，它具有希格斯对的量子数，但量子数上无法与轻子二重态区分。不存在左手反中微子。

The trinification model shown in Fig. 2 reduces to this model (plus additional non-chiral states) if one splits the second $U(3)$ to $U(2)\times U(1)$ and the third to $U(1)^3$.

若将第二个 $U(3)$ 拆分为 $U(2)\times U(1)$，第三个拆分为 $U(1)^3$，图 2 所示的三统一化模型就约化为该模型 (外加额外非手征态)。

Explicit realizations of this model have been found [18], but no global realizations with tadpole cancellation. The models presented in [23] are not explicit realizations, but hypothetical brane configurations with $x=0$ and $x=1$. Once x is fixed to these values, it is possible to get left-handed positrons as anti-symmetric tensors. Note that in classes (i) , (iii) , and (iv) , the value of x is either 0 or $\frac{1}{2}$.

该模型的显式构造已经在文献 [18] 中找到，但还没有带蝌蚪抵消的整体构造。文献 [23] 给出的并不是显式构造，而是带有 $x=0$ 和 $x=1$ 的假设膜构型。一旦将 x 固定为这些值，就可以得到作为反对称张量的左手正电子。注意在 (i) , (iii) 类和 (iv) 类中，x 的取值要么是 0，要么是 $\frac{1}{2}$。

Class (iii)

(iii) 类

This class can be viewed as a brane realization of an $SU(5)$ GUT model, with an extra $U(1)$. One takes an anti-symmetric tensor of $U(5)$ plus an anti-vector of $U(5)$, giving rise to the familiar $\mathbf{10}+\overline{\mathbf{5}}$. Now one may split the 5-stack into a 3-stack and a 2-stack. Physically, this may be realized by separating the two stacks by a certain amount in the compactified dimensions, or by using entirely different stacks with the same intersections. To construct a $U(5)$ GUT, we need a vector of $U(5)$, which is an open string with one endpoint on the $U(5)$ stack and its other endpoint on another brane, labeled c . This brane does not contribute the Y -charge and hence the simplest choice is an $O(1)$ brane. The explicit model in the table uses mechanism 3 to cancel the weak anomaly. This anomaly cancellation is inherited directly from the $SU(5)$ anomaly cancellation.

该类别可视为 $SU(5)$ 大统一理论的膜实现，带有一个额外的 $U(1)$。我们取 $U(5)$ 的一个反对称张量加上 $U(5)$ 的一个反矢量，得到我们熟知的 $\mathbf{10}+\overline{\mathbf{5}}$。接下来我们可以将 5 膜堆拆分为一个 3 膜堆和一个 2 膜堆。在物理上，这可以通过在紧致化维度中将两个膜堆分开一定距离，或是使用具有相同交点的完全不同的膜堆来实现。为构造一个 $U(5)$ 大统一理论，我们需要一个 $U(5)$ 的矢量，它是开弦，一个端点在 $U(5)$ 膜堆上，另一个端点在标记为 c 的另一块膜上。这块膜不贡献 Y 荷，因此最简单的选择是一块 $O(1)$ 膜。表格中的显式模型使用机制 3 抵消了弱反常。这种反常抵消直接继承自 $SU(5)$ 的反常抵消。

This class of models was first studied in [24], but without considering full tadpole cancellation. The latter problem was addressed in [25], but only examples with chiral exotics (15 of SU (5)) were found in this paper. The exact model in the table, with a brane group $U(3)\times U(2)\times O(1)$, has been found frequently in the search of [18], and there are even examples with full tadpole cancellation without any additional branes.

这类模型最早在文献 [24] 中被研究，但当时没有考虑完整的 tadpole 抵消。后者问题在文献 [25] 中得到处理，但该论文只找到了存在手征奇异态 (属于 SU (5) 的 15 维表示) 的例子。表格中这个膜群为 $U(3) \times U(2) \times O(1)$ 的精确模型，在文献 [18] 的搜索中被频繁找到，甚至存在不需要添加额外膜就能实现完整 tadpole 抵消的例子。

We anticipate that $SU(5)$ GUTs based on D-branes suffer from the absence of a top quark Yukawa coupling at the perturbative level. This problem is overcome in non-perturbative realizations of such models as described in section "F-Theory Model Building."

我们预计，基于 D 膜的 $SU(5)$ 大统一理论在微扰层面缺少顶夸克汤川耦合。正如"F 理论模型构建"一节所述，该问题可以在非微扰实现中得到解决。

Class (iv)

(iv) 类

This class can be made anomaly-free on the weak brane by using mechanism 3. In this case the anti-symmetric tensor is a left-handed antineutrino. The c brane anomaly cancels because the symmetric anti-tensor contributes -5, and the strings producing d^c and L have a contribution $3 + 2 = 5$.

这类模型可通过机制 3 在弱弦上消除反常，在此情形下反对称张量对应左手反中微子。c 弦反常得以抵消，因为对称反对张量贡献-5，产生 d^c 和 L 的弦贡献为 $3 + 2 = 5$ 。

This class contains flipped $SU(5)$ models. In flipped $SU(5)\, d^c$ (as opposed to u^c) is realized using an anti-symmetric tensor. Flipped $SU(5)$ requires an additional $U(1)$, and that $U(1)$ is realized here as a linear combination of the $U(1)$ from brane c and the phase factor $U(1)$ of $U(5)$. Orientifold models of this kind were first studied in [26], but also in the flipped $SU(5)$ case, the first examples found had chiral exotics. Explicit examples of the spectrum shown in the table have been found in the search of [18]. There are even examples of full tadpole cancellation without a hidden sector.

这一类包含翻转 $SU(5)$ 模型。在翻转 $SU(5)\, d^c$ 中 (与 u^c 不同)，是利用反对称张量实现的。翻转 $SU(5)$ 需要额外的 $U(1)$ ，此处 $U(1)$ 实现为来自膜 c 的 $U(1)$ 与 $U(5)$ 的相位因子 $U(1)$ 的线性组合。这类定向模模型最早在文献 [26] 中被研究，但即便是在翻转 $SU(5)$ 的情形下，最早找到的例子也存在手征奇异态。文献 [18] 的搜索中已经找到了表中所示能谱的明确例子，甚至存在不存在隐 sector 仍满足完全 tadpole 抵消的例子。

Yukawa Couplings

汤川耦合

The quarks and leptons discussed above must all acquire a mass from a three-point coupling with a Higgs boson. The latter should be present in the light spectrum. It must be a weak doublet, hence a bi-fundamental open string with one end on the weak brane. Let us call the brane with the other endpoint the Higgs brane.

前文讨论的夸克与轻子都必须通过与希格斯玻色子的三点耦合获得质量。希格斯玻色子应当存在于轻能谱中，它必须是弱作用二重态，因此是一端位于弱膜上的双基本开弦，我们将另一端所在的膜称为希格斯膜。

In perturbative open string theories, three-point couplings are represented by a disk diagram with three external lines, see (Fig. 5). For these couplings to exist, the three fields must be bi-fundamentals between branes (a,b), (b,c), and (c,a). If these branes are complex, the brane charges of each brane must cancel; in other words there must be an equal number of endpoints $\mathbf{N}$ and $\overline{\mathbf{N}}$ on each brane. These charges, the phase factors of the brane group $U(N)$, are conserved in string perturbation theory, even though the corresponding $U(1)$s may acquire a mass via the Stückelberg mechanism. The latter fact means that there are non-perturbative effects, generated by instantons, which break those symmetries, but these symmetries remain as global symmetries in perturbation theory, analogously to the baryon number in the Standard Model.

在微扰开弦理论中，三点耦合由带有三条外线的圆盘图表示，参见图 5。这类耦合存在的前提是，三个场必须是膜 (a,b), (b,c) 和 (c,a) 之间的双基本场。如果这些膜是复膜，每个膜的膜荷必须抵消；换言之，每个膜上必须有相等数量的端点 $\mathbf{N}$ 和 $\overline{\mathbf{N}}$。这些电荷 (即膜群 $U(N)$ 的相位因子) 在弦微扰论中是守恒的，即便对应的 $U(1)$ s 可以通过施蒂克尔贝格机制获得质量。后一事实意味着，存在瞬子生成的非微扰效应会破缺这些对称性，但这些对称性在微扰论中仍然作为整体对称性存在，这与标准模型中的重子数类似。

Given this rule for three-point couplings, we can now compute the required Higgs representation for quarks by tensoring the quark doublet with each quark singlet, u or d. In classes (iii) and (iv), we see immediately that respectively the up and down Yukawa couplings do not exist perturbatively, because the quark singlets are realized as anti-symmetric tensors, so we must rely on three vectors of $U(3)$ coupling to a singlet. This is fine in $SU(3)$, but not possible in $U(3)$, as first pointed out in [24].

根据三点耦合的这一规则，我们可以通过将夸克二重态与每个夸克单态 u 或 d 张量积，计算出夸克所需的希格斯表示。在 (iii) 类和 (iv) 类模型中，我们可以立即发现，上夸克和下夸克的汤川耦合分别无法在微扰论下存在，因为夸克单态实现为反对称张量，因此我们必须依赖 $U(3)$ 的三个矢量耦合到单态。这在 $SU(3)$ 中是可行的，但在 $U(3)$ 中不可能，这一点最早在文献 [24] 中指出。

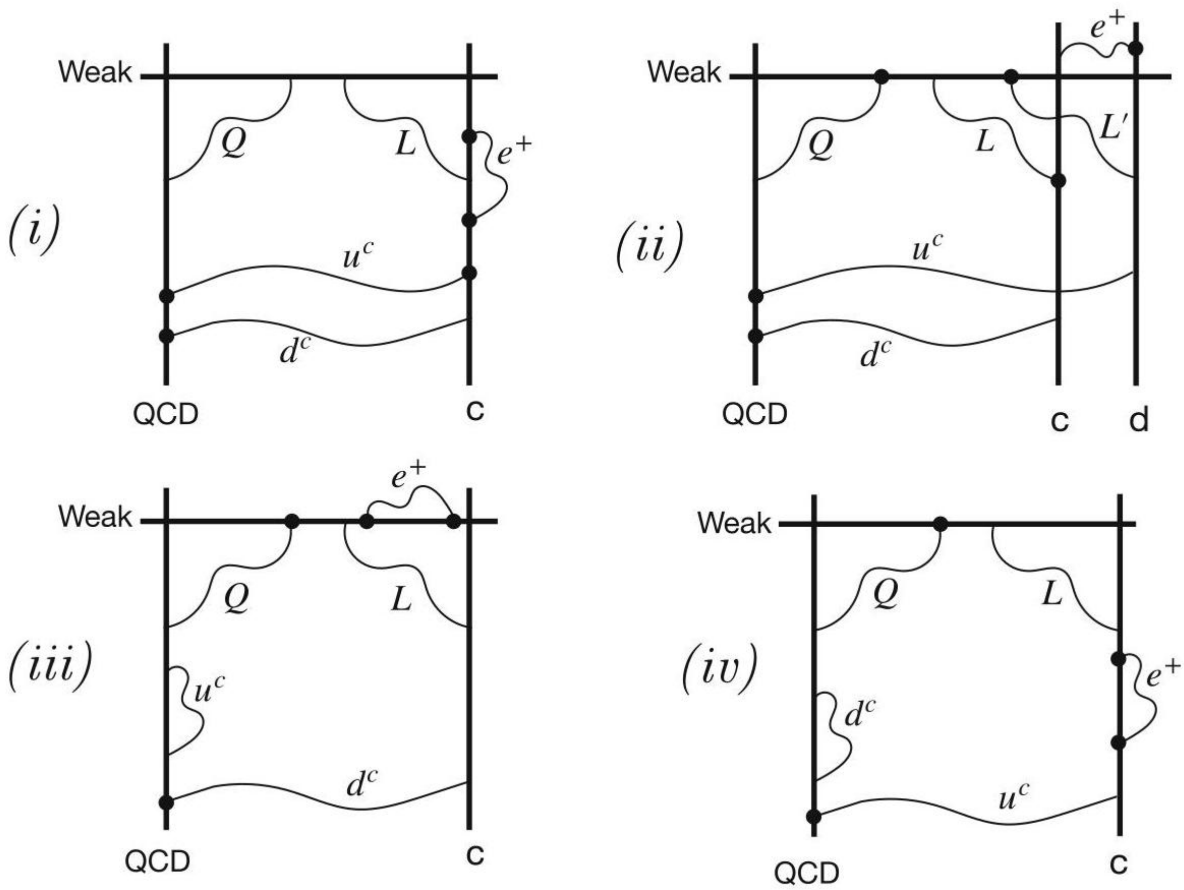
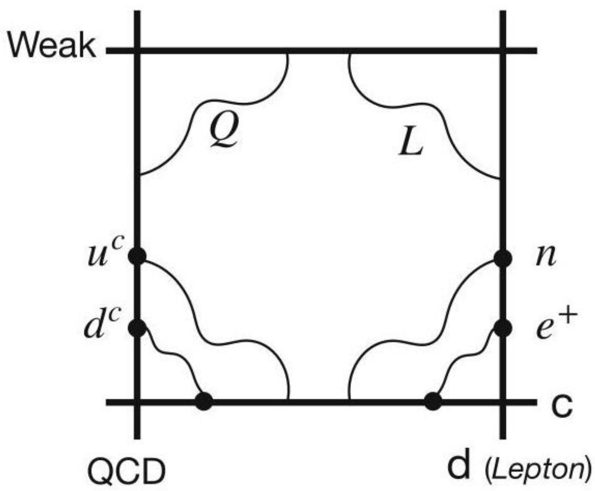


Fig. 3 The four basic classes from Table 3. Dots indicate a coupling to the orientifold image brane. L and L' are two possible assignments of the lepton doublet. In case (i) a weak $USp(2)$ group is assumed

> 图 3 表 3 中的四个基本类别。点表示对定向映像膜的耦合。L 和 L' 是轻子二重态的两种可能赋值。情形 (i) 假定存在一个弱 $USp(2)$ 群

Fig. 4 The ”Madrid” configuration, with a weak interaction group $USp(2)$

> 图 4 “马德里”构型，带有弱相互作用群 $USp(2)$



One may try to generate the missing Yukawa couplings non-perturbatively using instantons [27-32], or turn to F-theory models where they arise more naturally; see section ”GUT Constructions: Georgi-Glashow SU (5) GUTs”

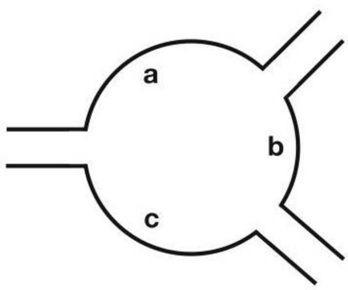
人们可以尝试利用瞬子非微扰地生成缺失的汤川耦合 [27-32]，也可以转向 F 理论模型，这类耦合在其中会更自然地出现；参见章节“GUT 构造: 乔治-格拉肖 SU(5) 大统一理论”

In classes (i) and (ii), a perturbative Yukawa coupling is possible, if one assigns the Higgs to one of the multiplets denoted L and L' in Fig. 3. Then there are a few more issues to worry about, weak brane and Higgs brane anomaly cancellation, and differences in weak $U(2)$ chirality for different families. In class (i) this is most easily dealt with by choosing the Madrid configuration (see Fig. 4) with $USp(2)$ as the weak brane group, and choosing brane c as the Higgs brane. Now one can choose $(0, \mathbf{N}, \mathbf{N}, 0) + \left(0, \mathbf{N}, \overline{\mathbf{N}}, 0\right)$ as the Higgs system. In class (ii) the fully orientable configuration discussed above already comes with a Higgs pair per family, but one of the Higgses has the same quantum numbers as the lepton doublet, which allows for undesirable couplings.

在 (i) 类和 (ii) 类模型中，如果将希格斯指定给图 3 中标记为 L 和 L' 的多重态之一，微扰汤川耦合就可以存在。此外还有几个问题需要处理: 弱膜与希格斯膜的反常抵消，以及不同代在弱 $U(2)$ 手征性上的差异。对于 (i) 类，最容易处理的方式是选择马德里构型 (参见图 4)，以 $USp(2)$ 作为弱膜群，选择膜 c 作为希格斯膜，然后可以选取 $(0, \mathbf{N}, \mathbf{N}, 0) + \left(0, \mathbf{N}, \overline{\mathbf{N}}, 0\right)$ 作为希格斯系统。对于 (ii) 类，前文讨论的完全可定向构型本身每个代就带有一个希格斯对, 但其中一个希格斯与轻子二重态具有相同的量子数，会产生不期望的耦合。

Fig. 5 Three-point open string coupling diagram

图 5 三点开弦耦合图



The discussion of lepton Yukawas goes along the same lines for the latter two models. Indeed, in the models of Fig. 4, quarks and leptons play a symmetric role. This is not true in the fully orientable model, which lacks a right-handed neutrino. However, it is not likely that neutrino masses are generated by the Standard Model Higgs mechanism. It is usually assumed that there is a Majorana mass component with a different, and necessarily non-perturbative, origin, in order to understand the smallness of neutrino masses using the seesaw mechanism [27,28]. This, as well as many other aspects of Yukawa couplings in orientifold models, is beyond the scope of this chapter.

轻子汤川耦合的讨论在后两种模型中遵循相同的思路。事实上，在图 4 的模型中，夸克和轻子扮演着对称的角色，这在全可定向模型中并不成立，该模型不存在右手中微子。然而，中微子质量不太可能由标准模型希格斯机制产生。为了借助跷跷板机制 [27,28] 解释中微子质量很小的性质，通常假设中微子存在马约拉纳质量分量，该分量具有不同且必然非微扰的起源。这一点，以及 orientifold 模型中汤川耦合的许多其他方面，都超出了本章的讨论范围。

Type II Orientifolds

II 型定向轨形

In this section we describe D-brane model building in specific Type II compacti-fications. Our framework will be Calabi-Yau (CY) threefold orientifold compact-ifications (Our discussion also applies to compactification backgrounds beyond Calabi-Yau metrics, like six-dimensional manifolds with $SU(3)$ or $SU(3) \times SU(3)$ structure, which feature a non-trivial warp factor and internal fluxes. We will however restrict ourselves to the CY case for simplicity.), at large volume and weak string coupling. This regime is where most of the model building ideas have been developed in string theory, because D-branes can be essentially treated as sub-manifolds in a compactification manifold X_6 , and as a result most of the quantities that specify the resulting 4d EFT have a simple topological or geometric realization. A large fraction of the intuition developed in this setup also applies to small-volume and strong-coupling compactifications, which will be dealt with in sections ”Rational Conformal Field Theories” and ”F-Theory Model Building,” respectively, and where also new model building features will arise.

本节我们将介绍在特定 II 型紧致化中构建 D 膜模型。我们的框架是卡拉比-丘 (CY) 三维定向轨形紧致化 (我们的讨论也适用于卡拉比-丘度规以外的紧致化背景，例如具有 $SU(3)$ 或 $SU(3) \times SU(3)$ 结构的六维流形，这类流形具有非平凡翘曲因子和内通量。但为简便起见，我们将讨论范围限定在 CY 情形)，讨论针对大体积和弱弦耦合区域。弦论中绝大多数模型构建思路都是在这个区域发展出来的，因为 D 膜本质上可以被视为紧致化流形 X_6 中的子流形，因此，描述最终得到的四维有效场论的大多数物理量都可以简单地通过拓扑或几何方式表示。在这套框架中建立的认知，大部分也适用于小体积和强耦合紧致化，这两类紧致化将分别在“有理共形场论”和“F 理论模型构建”两节中讨论，在这两类情形中也会出现新的模型构建特征。

Type IIA Orientifolds

IIA 型定向模

Let us consider Type IIA string theory on a background of the form $X_4 \times X_6$, where X_6 is a compact Calabi-Yau threefold X_6 , with Kähler two-form J and holomorphic three-form Ω_3 . To this background we apply an orientifold quotient generated by $\Omega(-1)^{F_L}\mathcal{R}$, where Ω is the worldsheet parity-reversal operator, F_L is the spacetime fermion number for the left-movers and $\mathcal{R}$ an anti-holomorphic involution of X_6 acting as $\mathcal{R}J = -J, \mathcal{R}\Omega_3 = -\overline{\Omega}_3$, respectively. The presence of $(-1)^{F_L}$ is important for the orientifold action to square to the identity. Performing the quotient has two main effects:

我们考虑背景形式为 $X_4 \times X_6$ 的 IIA 型弦论，其中 X_6 是紧致卡拉比-丘三维流形 X_6 ，具有凯勒二形式 J 和全纯三形式 Ω_3 。我们对该背景施加由 $\Omega(-1)^{F_L}\mathcal{R}$ 生成的定向模商，其中 Ω 是世界面宇称反演算符，F_L 是左动者的时空费米子数，$\mathcal{R}$ 分别是作用为 $\mathcal{R}J = -J, \mathcal{R}\Omega_3 = -\overline{\Omega}_3$ 的 X_6 的反全纯对合。$(-1)^{F_L}$ 的存在对于定向模作用平方等于单位元十分重要。执行该商操作有两个主要效应:

- It reduces the supersymmetry on the gravity sector from 4 d$\mathcal{N} = 2$ to $\mathcal{N} = 1$.

- 它将引力部分的超对称从 4 d$\mathcal{N} = 2$ 约化为 $\mathcal{N} = 1$ 。

- It introduces a set of O-planes at the fixed loci of $\mathcal{R}$ which, as explained in section ”D-Branes and Orientifolds,” are necessary ingredients for D-brane model building.

- 它在 $\mathcal{R}$ 的不动轨迹处引入了一组 O 平面，正如小节“D 膜与定向模”中解释的，这些是 D 膜模型构建的必要组分。

Since $\mathcal{R}$ is an anti-holomorphic involution, its fixed loci are given by a set of special Lagrangian three cycles of X_6 that we collectively denote as Π_{O6} , times the four non-compact dimensions X_4 . Thus, we have a set of O6-planes on $X_4 \times \Pi_{O6}$. In order to implement the strategy of section ”D-Brane Model Building: Generalities,” to this background, we add a set of space-time filling D-branes that give rise to the SM spectrum plus a hidden sector, and such that the RR tadpole cancellation conditions (6) with $p = 6$ are satisfied. The most natural option to build a vacuum is to consider D-branes that preserve the same supersymmetry as the bulk, because these are BPS objects that minimize their tension with respect to their RR charge, and then the cancellation of RR tadpoles implies that the bulk equations of motion are satisfied. If we focus on single D-branes, there are two types of objects that satisfy this condition. The first are D6-branes wrapped on a special Lagrangian three-cycle Π_3 , satisfying [33]

由于 $\mathcal{R}$ 是反全纯对合，它的不动轨迹由 X_6 的一组特殊拉格朗日三维闭链 (我们统一记为 Π_{O6}) 乘以四个非紧致维 X_4 给出。因此，我们在 $X_4 \times \Pi_{O6}$ 上得到一组 O6 平面。为了将小节“D 膜模型构建: 概述”的方案应用到该背景，我们引入一组填充时空的 D 膜，它们会产生标准模型谱加上一个隐藏区，并且满足带有 $p = 6$ 的 RR 蝌蚪抵消条件 (6)。构建真空最自然的选择是考虑保持和体空间相同超对称的 D 膜，因为这些是 BPS 物体，相对于它们的 RR 荷能最小化自身张力，此时 RR 蝌蚪抵消意味着体空间运动方程得到满足。如果聚焦于单个 D 膜，有两类物体满足该条件。第一类是包裹特殊拉格朗日三维闭链 Π_3 的 D6 膜，满足 [33]

$$\mathcal{F} + iJ = 0, \quad \text{and} \quad \operatorname{Im}\Omega_3 = 0, \tag{11}$$

where

其中

$$\mathcal{F} = B + \frac{\ell_s^2}{2\pi}F \tag{12}$$

is the gauge-invariant D-brane worldvolume field strength, with $\ell_s = 2\pi\sqrt{\alpha'}$ the string length. In all these expressions, bulk p -forms like B, J , and Ω_3 are implicitly pulled back to the D-brane worldvolume, in this case to the three-cycle Π_3 . That is, the D6-brane wraps a special Lagrangian three-cycle with the same calibration phase as Π_{O6} , along which it hosts a flat gauge bundle. These two conditions are referred to as F-flatness and D-flatness conditions in the literature, because if they are not met, there will be an uncancelled D-term or F-term in the 4d EFT gauge sector, respectively. The second kind of object are D8-branes wrapped on coisotropic five-cycle Ξ of X_6 [34], characterized by the BPS conditions:

是规范不变的 D 膜世界面场强，$\ell_s = 2\pi\sqrt{\alpha'}$ 为弦长。在所有这些表达式中，像 B, J 、Ω_3 这样的体 p 形式都被隐式拉回至 D 膜世界体积，在本例中就是拉回至三维闭链 Π_3 。也就是说，D6 膜包裹的特殊拉格朗日三维闭链具有和 Π_{O6} 相同的校准相位，膜上承载平坦规范丛。这两个条件在文献中被称为 F 平坦性条件和 D 平坦性条件: 如果不满足，四维有效场论的规范部分会分别存在未抵消的 F 项或 D 项。第二类物体是包裹 X_6 的余迷向五闭链 Ξ 的 D8 膜 [34]，其 BPS 条件为:

$$(\mathcal{F} + iJ)^2 = 0, \quad \text{and} \quad \operatorname{Im}\Omega_3 \wedge \mathcal{F} = 0, \tag{13}$$

which can again be interpreted as F-flatness and D-flatness conditions. While MSSM-like models have been built with coisotropic D8-branes [35] in the context of general CY geometries, it is technically difficult to describe these objects, and so in the Type IIA setting most of the activity has focused on building models based on D6-branes [8]. In the following we describe the main features of such models.

上述条件同样可以解释为 F 平坦性和 D 平坦性条件。尽管在一般卡拉比-丘几何背景下已经用余迷向 D8 膜构建出了类最小超对称标准模型 [35]，但这类物体的技术描述难度很高，因此在 IIA 框架下，绝大多数研究都聚焦于基于 D6 膜构建模型 [8]。下文我们将介绍这类模型的主要特征。

Intersecting D6-Brane Models

相交 D6 膜模型

Type IIA orientifold models were one of the last frameworks to be explored in the D-brane model building literature, but they quickly gained a central place in our current description of this topic. The reason is that the formula for the chiral index I_{ab} that appears in Table 1 is particularly simple, which gives us a lot of intuition about the chiral spectrum of these models. In particular, given N_a D6-branes wrapping a three-cycle $\Pi_3^a \subset X_6$ and a second stack of N_b D6-branes on Π_3^b , their chiral index reads:

IIA 型定向轨形模型是 D 膜模型构建文献中最晚被探索的框架之一，但它很快在我们目前对该主题的描述中占据了核心位置。原因在于表 1 中出现的手征指标公式 I_{ab} 格外简单，这让我们能对这些模型的手征谱形成大量直观认知。具体而言，给定 N_a 个 D6 膜包裹三维闭链 $\Pi_3^a \subset X_6$ ，另一组 N_b 个 D6 膜包裹 Π_3^b ，它们的手征指标为:

$$I_{ab} = [\Pi_3^a] \cdot [\Pi_3^b] \tag{14}$$

that is, the signed intersection number of the two three-cycles. More precisely, at each transverse intersection, one finds a left-handed $4d\mathcal{N} = 1$ chiral multiplet in a bi-fundamental representation, which is either $(N_a, \bar{N}_b)$ or $(\bar{N}_a, N_b)$ depending on the sign of the intersection, and (14) computes the net chiral spectrum in this sector. As anticipated, this is a topological invariant that only depends on the homology class of each three-cycle, or in other words of the RR charges of the branes. Finally, due to the orientifold symmetry, for each D6-brane stack wrapping Π_3^a , there is a similar number of D6-branes wrapping the orientifold image:

即两个三维闭链带符号的相交数。更准确地说，在每个横截交点处，会得到一个双基础表示下的左手征 $4d\mathcal{N}=1$ 手征多重态，根据相交符号的不同，它为 $(N_a,\bar{N}_b)$ 或 $(\bar{N}_a,N_b)$ ，而式 (14) 计算了该扇区的净手征谱。如前文所述，这是仅依赖每个三维闭链同调类的拓扑不变量，换言之，仅依赖膜的 RR 荷。最后，由于定向轨形对称性，对每个包裹 Π_3^a 的 D6 膜组，都有相同数量的 D6 膜包裹其定向轨形像:

$$\Pi_3^{a'} = \mathcal{R}\Pi_3^a \tag{15}$$

Including these three cycles and their intersections and identifying them properly under the orientifold action, one arrives at the spectrum of Table 1. In particular, at transverse intersections between Π_3^a and $\Pi_3^{a'}$ that are not on top of Π_{O6} , we get adjoint $\mathbf{Adj}=\mathbf{S}+\mathbf{A}$ representations of $U(N_a)$, while for those on top of Π_{O6} , one either gets a symmetric or an anti-symmetric representation.

将这些三维闭链及其相交纳入考虑，并在定向轨形作用下正确标识它们，就可以得到表 1 的能谱。具体而言，在 Π_3^a 与 $\Pi_3^{a'}$ 不位于 Π_{O6} 之上的横截交点处，我们得到 $U(N_a)$ 的伴随 $\mathbf{Adj}=\mathbf{S}+\mathbf{A}$ 表示，而对于位于 Π_{O6} 之上的交点，得到的要么是对称表示，要么是反对称表示。

In this setup we can also specify those stacks of N D6-branes that realize either an $SO(N)$ or $USp(N)$ gauge group. As mentioned in section "D-Branes and Orientifolds," these are D-brane sectors that are left invariant under the orientifold action. More precisely they are realized by D6-branes on three cycles satisfying the property $\Pi_3^a = \Pi_3^{a'}$. There are essentially two kinds of such three cycles: those that are left invariant point-wise and those that are only invariant as a set. Typically, the first kind gives rise to the real gauge group $SO(N)$ and the second one, which requires an even number of D6-branes, to $USp(N)$ [36].

在该框架中，我们还可以确定那些实现了 $SO(N)$ 或 $USp(N)$ 规范群的 N D6 膜组。正如“D 膜与定向轨形”一节所述，这些是在定向轨形作用下保持不变的 D 膜扇区。更准确地说，它们由满足性质 $\Pi_3^a = \Pi_3^{a'}$ 的三维闭链上的 D6 膜实现。这类三维闭链主要有两种: 逐点不变的闭链，以及仅作为集合不变的闭链。通常，第一种产生实规范群 $SO(N)$ ，第二种需要偶数个 D6 膜，产生 $USp(N)$ [36]。

The last piece of data needed to realize the content of Table 1 are those $U(1)$ factors that remain massless after the generalized Green-Schwarz mechanism and in particular all the $B\wedge F$ couplings have been taken into account. In the absence of an orientifold projection, these are the combinations $\sum_a c_a U(1)_a$ with $c_a\in\mathbb{Z}$ and such that the homology class $\sum_a c_a N_a[\Pi_3^a]$ is trivial in $H_3(X_3,\mathbb{Z})$. In the presence of O6-planes, some of the RR fields mediating the Green-Schwarz mechanism are projected out, and only the weaker condition

实现表 1 内容所需的最后一组数据是，经过广义格林-施瓦茨机制后仍保持无质量的 $U(1)$ 因子，尤其是在所有 $B\wedge F$ 耦合都被考虑在内之后。不存在定向轨形投影时，这些是满足 $c_a\in\mathbb{Z}$ 且同调类 $\sum_a c_a N_a[\Pi_3^a]$ 在 $H_3(X_3,\mathbb{Z})$ 中平凡的组合 $\sum_a c_a U(1)_a$ 。存在 O6 平面时，部分传递格林-施瓦茨机制的 RR 场被投影出去，仅剩下更弱的条件

$$\sum_a c_a N_a \left([\Pi_3^a] - \left[\Pi_3^{a'}\right]\right) = 0 \tag{16}$$

needs to be imposed [37]. Those combinations that do not satisfy (16) acquire a mass via a Stückelberg mechanism, but they remain as perturbative global symmetries that are only broken by non-perturbative effects. As discussed in section "Yukawa Couplings," they constrain the magnitude of those couplings that are not invariant under them, like certain Yukawa couplings, which can only be generated non-perturbatively $[27-30,32]$. Finally, if there are combinations of the form (16) with g.c.d $\{c_a\} = 1$ which are $2k$ multiples of some non-trivial element of $H_3(X_3, \mathbb{Z})$, the corresponding massive $U(1)$ contains a $\mathbb{Z}_k$ subgroup that is an exact gauge symmetry. This will prevent the appearance of certain couplings even at the non-perturbative level [38].

需要被满足 [37]。不满足式 (16) 的组合会通过施蒂克尔贝格机制获得质量，但它们仍作为微扰整体对称性存在，仅被非微扰效应破缺。正如“汤川耦合”一节中讨论的，它们会约束那些在该对称性下不不变的耦合，例如某些汤川耦合只能通过非微扰方式生成 $[27-30,32]$ 。最后，如果存在形如式 (16) 的组合，其最大公约数 $\{c_a\} = 1$ 是 $2k$ 中某个非平凡元素的倍数，属于 $H_3(X_3, \mathbb{Z})$ ，则对应的有质量 $U(1)$ 包含一个子群 $\mathbb{Z}_k$ ，该子群是精确规范对称性。这会阻止特定耦合在非微扰能级甚至也无法出现 [38]。

With these ingredients one may already start discussing explicit examples of intersecting D6-brane models that realize the model building philosophy of section "D-Brane Model Building: Generalities." In general, the topological data that one needs are the lattice $H_3(X_6, \mathbb{Z})$, the action of the involution $\mathcal{R}$ on it, the class $[\Pi_{O6}]$, as well as the intersection product (14). Further data that are important for D6-brane model building are those classes $[\Pi] \in H_3(X_6, \mathbb{Z})$ with $\mathrm{Im} \int_\Pi \Omega_3 = 0$ that contain special Lagrangian representatives. Determining them is the hardest part of the problem, and hence a large fraction of Type IIA orientifold model building is performed in simple geometries like toroidal orbifolds.

有了这些要素，我们就可以开始讨论相交 D6 膜模型的具体实例，实现“D 膜模型构建: 概述”一节的模型构建思路。一般来说，我们需要的拓扑数据包括格点 $H_3(X_6, \mathbb{Z})$ ，对合 $\mathcal{R}$ 在其上的作用，类 $[\Pi_{O6}]$ ，以及式 (14) 的相交乘积。对 D6 膜模型构建而言，另一类重要数据是包含特殊拉格朗日代表的类 $[\Pi] \in H_3(X_6, \mathbb{Z})$ ，满足条件 $\mathrm{Im} \int_\Pi \Omega_3 = 0$ 。确定这些类是该问题最困难的部分，因此 IIA 型定向模模型构建大多在环面 orbifold 这类简单几何中完成。

A Simple Model

一个简单模型

Let us illustrate the general strategy of section "D-Brane Model Building: Generalities" in a simple MSSM-like model. We focus on building a Class (i) model in the classification of section "The Simplest Examples," following [39,40]. The first step is to specify the four sets of D6-branes that host the MSSM-like spectrum, which we do as in Table 4. We consider two stacks of D6-branes (b and c) invariant under the orientifold projection and such that the gauge group for each of them is $USp(2) \simeq SU(2)$. For the remaining two stacks (a and d) , we choose Π_3^a and Π_3^d to lie in the same homology class, up to a torsion element in $H_3(X_6, \mathbb{Z})$. This implies that they will have the same intersection number with any other three-cycle and that $U(1)_{B-L} = \frac{1}{3}U(1)_a - U(1)_d$ will remain massless. With these choices we only need to specify four intersection numbers, $I_{ab} = -I_{ac} = 3$ and $I_{aa'} = I_{aO} = 0$, in order to reproduce the model of Fig. 4, or more precisely a left-right extension of the SM gauge group. If in addition $\Pi_3^a = \Pi_3^d$, one arrives at a Pati-Salam model, whose spectrum is specified in the upper part of Table 6. This case is particularly simple to realize in a concrete CY geometry because three cycles hosting USp gauge groups automatically satisfy the supersymmetry condition (11), and so it only remains to verify that the cycles $\Pi_3^a = \Pi_3^d$ also preserve supersymmetry at some point in the CY moduli space.

我们将通过一个类似最小超对称标准模型 (MSSM) 的简单模型，演示“D 膜模型构建: 概述”一节的通用思路。我们遵循文献 [39,40]，按照“最简实例”一节的分类，聚焦构建 (i) 类模型。第一步是确定承载类 MSSM 谱的四组 D6 膜，如表 4 所示。我们考虑两组满足 orientifold 投影不变的 D6 膜堆 (b 和 c)，每组的规范群均为 $USp(2) \simeq SU(2)$ 。对于剩余两组 D6 膜堆 (a and d)，我们选取 Π_3^a 和 Π_3^d ，允许相差 $H_3(X_6, \mathbb{Z})$ 中的一个挠元，二者处于相同同调类。这意味着它们和任意其他三周期的相交数都相同，且 $U(1)_{B-L} = \frac{1}{3}U(1)_a - U(1)_d$ 保持无质量。基于这些选择，我们仅需确定四个相交数 $I_{ab} = -I_{ac} = 3$ 和 $I_{aa'} = I_{aO} = 0$ ，即可重现图 4 的模型，更准确地说，得到标准模型规范群的左右对称扩展模型。如果额外满足 $\Pi_3^a = \Pi_3^d$ ，就得到帕蒂-萨拉姆模型，其谱列在表 6 的上半部分。这种情况在具体卡拉比-丘 (CY) 流形中尤其容易实现: 承载 USp 规范群的三周期自动满足超对称条件 (11)，因此只需验证 $\Pi_3^a = \Pi_3^d$ 对应的周期在 CY 模空间的某点处也保持超对称即可。

Table 4 Left-right model of intersecting D6-branes. Here $[\Pi_3^a] - [\Pi_3^d] \in \mathrm{Tor}\, H_3(X_6, \mathbb{Z})$

表 4 相交 D6 膜的左右对称模型。此处 $[\Pi_3^a] - [\Pi_3^d] \in \mathrm{Tor}\, H_3(X_6, \mathbb{Z})$

D6-brane content	$3\Pi_3^a$	Π_3^b	Π_3^c	Π_d
Gauge group	$SU(3) \times U(1)_a$	USp(2)	USp(2)	$U(1)_d$

To proceed with the construction of the model, one must specify the CY geometry. As noted, the simplest choices correspond to toroidal orbifolds and, in this case, to the orbifold $X_6 = (T^2 \times T^2 \times T^2)/\mathbb{Z}_2 \times \mathbb{Z}_2$ with cohomology $(h^{1,1}, h^{2,1}) = (51, 3)$, whose O6-plane quotient and D6-brane model building rules were worked out in [41, 42] . The simplicity of this geometry stems from the fact that the twisted sector only contains collapsed two cycles, and so $H_3(X_6, \mathbb{Z})$ and its intersection product is essentially that of $T^2 \times T^2 \times T^2$. To specify a three-cycle class, one must provide a one-cycle class on each T^2 factor that specify the following wrapping numbers:

要继续构建模型，必须确定 CY 几何。如前所述，最简单的选择是环面 orbifold，具体来说是上同调为 $(h^{1,1}, h^{2,1}) = (51, 3)$ 的 orbifold $X_6 = (T^2 \times T^2 \times T^2)/\mathbb{Z}_2 \times \mathbb{Z}_2$ ，其 O6 平面商空间与 D6 膜模型构建规则已在 [41, 42] 中推导完成。这种几何的简洁性源于: 扭曲扇区仅包含坍缩的二周期，因此 $H_3(X_6, \mathbb{Z})$ 及其相交积本质上与 $T^2 \times T^2 \times T^2$ 的相交积相同。要确定一个三周期类，需要为每个 T^2 因子指定一个一周期类，从而得到如下缠绕数:

$$[\Pi_3^a] = (n_a^1, m_a^1) \times (n_a^2, m_a^2) \times (n_a^3, m_a^3), \; n_a^i, m_a^i \in \mathbb{Z}, \tag{17}$$

and then the intersection number between two three cycles is given by

两个三周期之间的相交数由下式给出

$$I_{ab} = [\Pi_3^a] \cdot [\Pi_3^b] = \left(n_a^1 m_b^1 - m_a^1 n_b^1\right) \times \left(n_a^2 m_b^2 - m_a^2 n_b^2\right) \times \left(n_a^3 m_b^3 - m_a^3 n_b^3\right). \tag{18}$$

In a toroidal orientifold geometry, such three cycles must be accompanied by their images under the orientifold group. In the case at hand, the $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold group, the wrapping numbers (17) are mapped to themselves, and the smallest or fractional D-brane objects correspond to two copies of (17) in certain locations. The orientifold image is dictated by the anti-holomorphic involution $\mathcal{R} : z^i \to \bar{z}^i$, where z^i is the complex coordinate on the i th T^2 . That this involution is a symmetry of the internal metric restricts the moduli space of complex structures, with one possibility being that each T^2 is rectangular. In that case, which we will assume in the following, the orientifold image of (17) is

在环面定向扭结几何中，这类三周期必须伴随其在定向扭结群下的镜像。就当前情况而言，对于 $\mathbb{Z}_2 \times \mathbb{Z}_2$ 轨形群，缠绕数 (17) 映射到自身，最小即分数 D-膜对象对应于特定位置处两份 (17) 的拷贝。定向扭结镜像由反全纯对合 $\mathcal{R} : z^i \to \bar{z}^i$ 决定，其中 z^i 是第 i 个 T^2 上的复坐标。该对合是内禀度规的对称性，这一点限制了复结构的模空间，其中一种可能是每个 T^2 都是矩形的。我们接下来将采用这种情况，此时 (17) 的定向扭结镜像为

$$\left[\Pi_3^{a'}\right] = \left(n_a^1, -m_a^1\right) \times \left(n_a^2, -m_a^2\right) \times \left(n_a^3, -m_a^3\right), \tag{19}$$

and the O6-plane class is given by

且 O6-平面类由下式给出

$$[\Pi_{O6}] = 4[(1,0) \times (1,0) \times (1,0) + (1,0) \times (0,1) \times (0,-1) \tag{20}$$

$$+ (0,1) \times (1,0) \times (0,-1) + (0,1) \times (0,-1) \times (1,0)], \tag{21}$$

in fractional three-cycle units. A stack of D6-branes with the wrapping numbers of one of the components of (21) hosts a USp gauge group. Because such wrapping numbers are invariant under the full orientifold group, one only needs two D6-branes in the covering space to host a $USp(2)$ gauge group.

单位为分数三周期。一堆具有 (21) 其中一个分量缠绕数的 D6-膜承载一个 USp 规范群。由于这类缠绕数在整个定向扭结群下不变，覆盖空间中仅需要两张 D6-膜就能承载一个 $USp(2)$ 规范群。

Table 5 Explicit $\left(T^2 \times T^2 \times T^2\right)/\mathbb{Z}_2 \times \mathbb{Z}_2$ orientifold model realizing the MSSM-like model of Table 4. D6-brane multiplicities are given in fractional three-cycle units

表 5 实现表 4 类最小超对称标准模型的显式 $\left(T^2 \times T^2 \times T^2\right)/\mathbb{Z}_2 \times \mathbb{Z}_2$ 定向扭结模型。D6-膜多重度以分数三周期为单位给出

N_α	(n^1_α, m^1_α)	(n^2_α, m^2_α)	(n^3_α, m^3_α)
$N_a = 3+1$	(1,0)	(3,1)	(3, -1)
$N_b = 1$	(0,1)	(1,0)	(0, -1)
$N_c = 1$	(0,1)	(0,-1)	(1,0)
$N_{h_1} = 1$	(-2,1)	(-3,1)	(-4,1)
$N_{h_2} = 1$	(-2,1)	(-4,1)	(-3,1)
$N_f = 20$	(1,0)	(1,0)	(1,0)

Table 6 Spectrum of the model of Table 5 after the D6-brane recombination $h_1 + h_2' \to h$, with the subindex indicating the charge under $U(1)' = \frac{1}{3}U(1)_a + 2U(1)_h$. There is no chiral matter under the gauge group $USp(40)$, which is a hidden sector

表 6 D6-膜复合 $h_1 + h_2' \to h$ 后表 5 中模型的能谱，下标表示 $U(1)' = \frac{1}{3}U(1)_a + 2U(1)_h$ 下的电荷。规范群 $USp(40)$ 下无手征物质，该群是一个暗 sector

Sector	Matter	$SU(4) \times SU(2) \times SU(2) \times U(1)' \times USp(40)$
(ab)	F_L	$3(4,2,1)_{1/3}$
(ac)	F_R	$3(4,1,2)_{-1/3}$
(bc)	H	$(1,2,2)_0$
(bh)		$2(1,2,1)_2$
(ch)		$2(1,1,2)_{-2}$

With these ingredients one may already build an explicit model, by providing the wrapping number content of Table 5. Notice that the upper part of the table realizes the intersection numbers needed for the Pati-Salam spectrum, and one finds in addition a minimal Higgs sector with a nonvanishing μ -term; see Table 6. The lower part of Table 5 is the one needed to satisfy the RR tadpole cancellation conditions (6). In principle this sector introduces an additional gauge group $U(1)' \times USp(40)$, with $U(1)' = \frac{1}{3}U(1)_a + 2\left[U(1)_{h_1} - U(1)_{h_2}\right]$, which we would like to treat as a hidden sector of the theory. However, the $U(1)$ branes h_1 and h_2 intersect with those of the Pati-Salam sector, generating chiral exotics that naively prevent us from doing so; see [39, 40] for the resulting spectrum.

利用这些要素，我们已经可以通过给出表 5 的缠绕数内容构造一个显式模型。注意该表的上半部分实现了帕蒂-萨拉姆能谱所需的相交数，此外还能得到带有非零 μ 项的最小希格斯 sector，参见表 6。表 5 的下半部分是满足 RR 蝌蚪抵消条件 (6) 所需的部分。原则上该部分会引入额外的规范群 $U(1)' \times USp(40)$ ，带有 $U(1)' = \frac{1}{3}U(1)_a + 2\left[U(1)_{h_1} - U(1)_{h_2}\right]$ ，我们希望将其作为理论的暗 sector。但 $U(1)$ 膜 h_1 和 h_2 与帕蒂-萨拉姆 sector 的膜相交，会产生手征奇异态，直观来看这会让我们无法将上述部分视作暗 sector；所得能谱参见 [39, 40] 。

Nevertheless, one of the main advantages of the model building strategy outlined in section "D-Brane Model Building: Generalities" is that this kind of chiral exotics is easily avoidable. Indeed, because the particle content of the Pati-Salam sector cancels all cubic $SU(N)$ anomalies by itself (including those for $N = 2$ and $N = 1$, which in this particular case are trivial), the chiral exotics that arise from additional D-brane sectors appear as vector-like from the viewpoint of the visible sector gauge group. In practice, this means that there is some direction in moduli space that allows us to get rid of such chiral exotics, as it is the case in this example. More precisely, by moving in the complex structure moduli space of the first T^2 , one can induce a tachyon in the bi-fundamental of $U(1)_{h_1} \times U(1)_{h_2}$ via a D-term potential; see below. Condensation of this tachyon corresponds to the D-brane recombination process $h_1 + h_2' \to h$, which does not affect the low-energy gauge group, but greatly simplifies the chiral spectrum. The final result is given in Table 6, where one can see that the lower part of the spectrum is not chiral under the Pati-Salam gauge group. Given the simplicity of this construction, this is quite an encouraging result, which one may hope to improve by exploring more general setups. Indeed, applying the same approach to more involved toroidal orbifolds, general Calabi-Yau manifolds, and RCFT models yields even more realistic models.

尽管如此，在“D 膜模型构建: 概述”一节中提出的这种模型构建策略的核心优势之一，就是这类手征外态很容易规避。事实上，由于帕蒂-萨拉姆扇区的粒子谱本身抵消了所有三次 $SU(N)$ 反常 (包括 $N = 2$ 和 $N = 1$ 对应的反常，在这个特例中这些反常本身就是平凡的)，来自额外 D 膜扇区的手征外态在可见层规范群看来都是矢量类的。这在实际中意味着，模空间中存在某个方向可以让我们消除这类手征外态，本例正是如此。更准确地说，通过移动第一个 T^2 的复结构模空间，我们可以通过 D 项势在 $U(1)_{h_1} \times U(1)_{h_2}$ 的双基本表示中诱导出快子，详见下文。该快子凝聚对应 D 膜重组过程 $h_1 + h_2' \to h$ ，这一过程不会影响低能规范群，但能大幅简化手征谱。最终结果见表 6，从中可以看出，谱的下半部分在帕蒂-萨拉姆规范群下不再是手征的。鉴于该构造十分简单，这已经是相当令人鼓舞的结果，我们有望通过探索更一般的构型进一步优化结果。事实上，将这套方法应用到更复杂的环面轨形、一般卡拉比-丘流形以及有理共形场论模型中，可以得到更贴近现实的模型。

Further EFT Features

有效场论进一步特征

Besides a reasonable chiral spectrum, a realistic model must display a set of couplings and a non-chiral light spectrum that are compatible with the MSSM or extensions thereof. In the sequel we outline the computation of these additional features in the context of intersecting D6-brane models.

除了合理的手征谱之外，realistic 模型还必须拥有一组与最小超对称标准模型或其扩展兼容的耦合与非手征轻谱。下文我们将在相交 D6 膜模型的框架下概述这些额外特征的计算方法。

The simplest quantity to consider is the gauge kinetic function associated with each stack of D6-brane. It reads:

最简单的可计算量是对应每堆 D6 膜的规范动力学函数，其形式为:

$$f_a^{\mathrm{D6}} = \frac{1}{2\pi\ell_s^3}\int_{\Pi_3^a} e^{-\phi}\,\mathrm{Re}\,\Omega_3 + iC_3, \tag{22}$$

from where one can compute the gauge couplings of the non-abelian and abelian gauge factors. In the latter case, there will generically be a kinetic mixing with bulk $U(1)$ gauge symmetries, if present [43].

由此我们可以计算非阿贝尔与阿贝尔规范因子的规范耦合。对于阿贝尔规范因子, 若存在 bulk $U(1)$ 规范对称性，通常会产生动力学混合 [43]。

One may also consider the presence of light or massless non-chiral particles in the D-brane sector, such as D-brane moduli that appear as $\mathcal{N} = 1$ multiplets in the adjoint. A stack of N D6-branes wrapping a special

Lagrangian three-cycle Π_3 has $b_1(\Pi)$ deformations of its embedding that preserve the special Lagrangian condition, as it follows from McLean's theorem. These are complexified by the same amount of Wilson lines, giving rise to $b_1(\Pi)\, \mathcal{N} = 1$ chiral multiplets in the adjoint representation of $SU(N)$. Aiming to build models without adjoints leads us to either consider three cycles with $b_1(\Pi) = 0$ or D6-branes with deformations that are fixed by an F-term potential. There are two sources for the latter. The first source are elements of $H_1(\Pi, \mathbb{Z})$ dual to two cycles of Π_3 that are non-trivial in $H_2(X_6, \mathbb{Z})$ [44]. The second one is the superpotential generated by disc worldsheet instantons ending on one cycles of Π_3 [45]. While this second source in principle affects all adjoint fields, it is generically expected to give them exponentially suppressed masses in the large volume regime in which we are working. In general, D-brane adjoints redefine the 4d holomorphic variables that arise from bulk moduli upon dimensional reduction. In this case D6-brane moduli redefine the CY complex structure moduli (see, e.g., [46-48]), and their mass spectrum should be treated in the broader framework of moduli stabilization. Such complex structure moduli enter the D-flatness condition for D6-branes, and can induce Fayet-Iliopoulos terms which break supersymmetry and may trigger D6-brane recombination [49].

我们也可以考虑 D 膜 sector 中存在轻的或无质量的非手征粒子，比如伴随表示中以 $\mathcal{N} = 1$ 多重态形式出现的 D 膜模。根据麦克莱恩定理，一堆缠绕特殊拉格朗日 3-cycle Π_3 的 N D6 膜，存在保持特殊拉格朗日条件的 $b_1(\Pi)$ 个嵌入形变。这些形变可被同等数量的威尔逊线复化，最终在 $SU(N)$ 的伴随表示中生成 $b_1(\Pi)\, \mathcal{N} = 1$ 个手征多重态。若想要构建无伴随场的模型，我们要么选择本身带 $b_1(\Pi) = 0$ 的 3-cycle，要么选择形变被 F 项势固定的 D6 膜。后者存在两种来源: 第一种来源是 $H_1(\Pi, \mathbb{Z})$ 中，在 $H_2(X_6, \mathbb{Z})$ 内非平凡且对偶于 Π_3 的 2-cycle 的元素 [44]；第二种来源于终止于 Π_3 的 1-cycle 的圆盘世界面瞬子生成的超势 [45]。原则上第二种来源会影响所有伴随场，但一般认为在我们工作的大体积区域，它给这些场带来的质量是指数压低的。一般来说，D 膜伴随场会重新定义维约化后从体模得到的 4 维全纯变量。在本情形中，D6 膜模会重新定义卡拉比-丘复结构模 (例如参见 [46-48])，其质量谱需要放在模稳定的更广泛框架下讨论。这类复结构模会进入 D6 膜的 D 平坦性条件，可以诱发费耶特-伊利亚 opoulos 项，该项会破坏超对称，并可能触发 D6 膜重组 [49]。

Besides adjoint masses, worldsheet instantons with the topology of a disc are a source for mass terms for vector-like pairs that arise from the transverse intersections of a pair of D6-branes, as illustrated in Fig. 6, as well as for Yukawa couplings [7]. These two quantities have a direct interpretation in terms of Kontsevich's homological mirror symmetry conjecture, and their computation is a rich mathematical subject of research. While they are difficult to compute in general (see [50] for recent progress), in simple examples like toroidal orbifold models, one can perform the computation quite explicitly, showing for instance that the Pati-Salam model described above leads to Yukawa couplings of rank one [21].

除了伴随场质量外，圆盘拓扑的世界面瞬子还会为一对 D6 横截相交产生的类矢量对提供质量项 (如图 6 所示)，同时也能提供汤川耦合 [7]。这两个量可以直接通过康采维奇的同调镜对称猜想解释，它们的计算是一个非常活跃的数学研究课题。尽管一般情况下它们很难计算 (最新进展参见 [50])，但在环面 orbifold 模型这类简单例子中，我们可以非常明确地完成计算，例如可以证明上文描述的帕蒂-萨拉姆模型给出秩为一的汤川耦合 [21]。

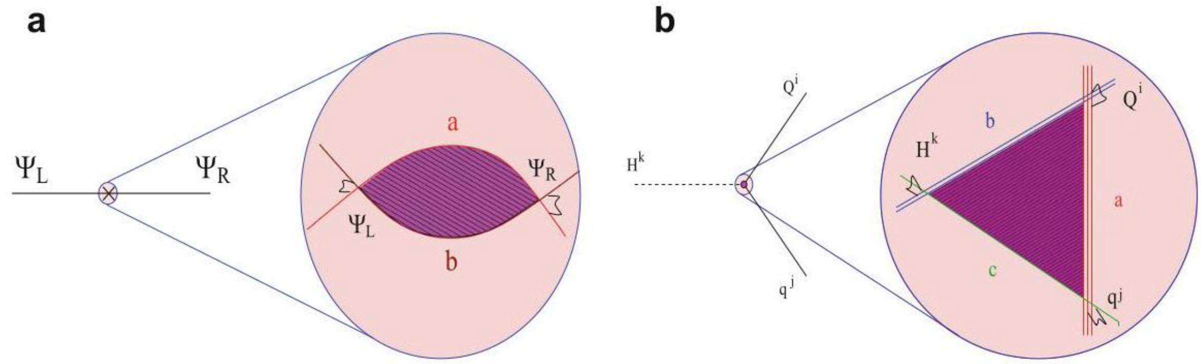


Fig. 6 Worldsheet instantons as generators of (a) masses for vector-like pairs and (b) Yukawa couplings. (Figures taken from [10])

图 6 世界面瞬子作为 (a) 类矢量对质量和 (b) 汤川耦合的生成源。(图取自 [10])

What worldsheet instantons cannot generate are couplings that are forbidden by D-brane $U(1)$ symmetries that become massive due to a Stückelberg mechanism. In that case, couplings should be generated by D-brane instantons, which in this case are D2-brane instantons wrapping special Lagrangian three cycles of X_6 [32]. In typical models such couplings include right-handed neutrino masses and Yukawas forbidden by global $U(1)$ symmetries, as described in section "D-Brane Model Building: Generalities." Moreover, if any of these couplings is forbidden by the discrete gauge symmetries that are remnants of the massive $U(1)$s , they will not be generated even at the non-perturbative level. The model building challenge then resides in using this structure to forbid unwanted couplings and obtain those that are necessary phenomenologically, with the appropriate magnitude given by their suppression factors.

世界面瞬子无法生成的是那些被 D 膜 $U(1)$ 对称性禁戒的耦合，这些对称性会由于施蒂克尔贝格机制获得质量。这种情况下，耦合应当由 D 膜瞬子生成，具体来说是包裹 X_6 特殊拉格朗日三维闭链的 D2 膜瞬子 [32]。在典型模型中，这类耦合包含被整体 $U(1)$ 对称性禁戒的右手中微子质量和汤川耦合，正如“D 膜模型构建: 概述”一节所述。此外，如果这些耦合中有任何一个被作为有质量 $U(1)$s 残余的离散规范对称性禁戒，那么即使在非微扰层面它们也无法生成。因此模型构建的难点在于利用这一结构禁戒不必要的耦合，得到唯象学上所需的、被压低因子给出合适幅度的耦合。

Type IIB Orientifolds

IIB 型定向轨

We now turn to Type IIB string theory on the 10d background $X_4 \times X_6$, where again X_6 is taken to be a compact Calabi-Yau threefold X_6 , with Kähler two-form J and holomorphic three-form Ω_3 . There are two different kinds of orientifold projections which are compatible with a large compactification volume:

我们现在研究 10 维背景 $X_4 \times X_6$ 下的 IIB 型弦理论，其中约定 X_6 为紧致卡拉比-丘三 foliate X_6 ，具有凯勒二形式 J 和全纯三形式 Ω_3 。共有两种不同的定向投影与大紧致化体积相容:

$$\text{O3/O7 projection:}\quad \Omega(-1)^{F_L}\mathcal{R}\ \text{ such that }\ \mathcal{R}J = J,\ \mathcal{R}\Omega_3 = -\Omega_3\,,$$

$$\text{O5/O9 projection}:\ \Omega\mathcal{R}\ \text{ such that }\ \mathcal{R}J = J,\ \mathcal{R}\Omega_3 = \Omega_3\,,$$

where $\mathcal{R}$ is now a holomorphic involution of X_6 . As in the Type IIA case, this projection reduces the bulk supersymmetry from 4 d$\mathcal{N} = 2$ to $\mathcal{N} = 1$ and introduces a series of O-planes at the fixed loci of $\mathcal{R}$. The difference is that these fixed loci are even-dimensional submanifolds of X_6 . In the first projection, they are given by points or holomorphic four cycles, leading to O3 and/or O7-planes, while the second projection leaves invariant curves or the whole of X_6 . These holomorphic involutions are much better understood than their anti-holomorphic counterparts, which has resulted in the construction of models in geometries beyond toroidal orbifolds. The same remark applies to the space-time D-branes that host gauge interactions in these models, which now consist of D3, D5, D7, and D9-branes wrapping internal even-dimensional cycles Π_{p-3} of X_6 . The BPS conditions for a single D-brane of this kind read:

其中 $\mathcal{R}$ 是 X_6 的全纯对合。与 IIA 型情况一样，该投影将整体超对称从 4 d$\mathcal{N} = 2$ 约化为 $\mathcal{N} = 1$ ，并在 $\mathcal{R}$ 的不动点轨迹处引入一系列 O 平面。不同之处在于，这些不动轨迹是 X_6 的偶维子流形。第一种投影中，不动轨迹为点或全纯四循环，对应 O3 和/或 O7 平面，第二种投影则保持曲线或整个 X_6 不变。这类全纯对合比反全纯对应物研究得更充分，因此人们已经在环面 Orbifold 之外的几何中构造出了这类模型。同样的结论也适用于这些模型中承载规范相互作用的时空 D 膜，此处 D 膜由卷绕 X_6 内部偶维循环 Π_{p-3} 的 D3、D5、D7 和 D9 膜构成。这类单个 D 膜的 BPS 条件为:

$$\Pi_{p-3}\ \text{holomorphic},\ \mathcal{F}^{(2,0)} = 0,\quad \text{F-flatness}, \tag{23}$$

$$\operatorname{Im} e^{-i\theta} e^{\mathcal{F}+iJ}\sqrt{\hat{A}_{\Pi_{p-3}}} = 0,\quad \text{D-flatness}, \tag{24}$$

where $\theta = 0$ for the O3/O7-projection and $\theta = \pi/2$ in the O5/O9 projection and $\hat{A}$ is the A-roof genus of the tangent bundle of Π_{p-3} , encoding part of the D-brane curvature couplings. As a non-trivial worldvolume field strength is allowed, these objects must typically be understood as D3/D5/D7/D9 bound states [51] or, from a more mathematical perspective, as coherent sheaves [52,53]. These abelian BPS conditions can be generalized to non-abelian D-brane configurations that are allowed for stacks of several D-branes, featuring a non-abelian field strength $\mathcal{F}$ and/or a non-abelian D-brane worldvolume embedding.

其中 O3/O7 投影对应 $\theta = 0$, O5/O9 投影对应 $\theta = \pi/2$, $\hat{A}$ 是 Π_{p-3} 切丛的 A-顶亏格, 刻画了 D 膜曲率耦合的部分分量。由于世界 volume 允许存在非平庸场强，这类对象通常必须理解为 D3/D5/D7/D9 束缚态 [51]，或从更数学的角度理解为凝聚层 [52,53]。这些阿贝尔 BPS 条件可以推广到多个 D 膜叠放允许的非阿贝尔 D 膜构型，这类构型具有非阿贝尔场强 $\mathcal{F}$ 和/或非阿贝尔 D 膜世界 volume 嵌入。

Non-abelian D-brane configurations have mostly played a role in models with D9-branes, due to their analogy with heterotic compactifications. Due to this similarity, they will not be discussed here, nor will be models with O5/O9 projection. Instead, we will focus on models with O3/O7 projection, which display a set of features that are very representative of D-brane model building:

非阿贝尔 D 膜构型主要在含 D9 膜的模型中发挥作用，因为它们与杂化紧致化类似。由于这种相似性，本文不再讨论这类构型，也不讨论 O5/O9 投影的模型。相反，我们将聚焦 O3/O7 投影的模型，这类模型具有一系列非常能代表 D 膜模型构建的特征:

- The SM gauge group or its extension can be localized in a patch of the compact manifold X_6 . This simplifies its construction, which can be done in two steps: (i) first building a model in a local patch and (ii) embedding it into a compact manifold; see Fig. 7. This procedure is known as the bottom-up approach to model building [54], and it allows one to distinguish between those physical features that are only sensitive to local data and those that depend on global data of the construction.

- 标准模型规范群或其扩展可以定域在紧致流形 X_6 的一个局部补丁中。这简化了模型构建过程，可分为两步:(i) 先在局部补丁中构建模型；(ii) 将其嵌入紧致流形；见图 7。这一过程被称为自下而上的模型构建方法 [54]，它可以区分仅依赖局部数据的物理特征和依赖构造整体数据的物理特征。

- Due to this localization, and the fact that 4d gauge couplings are controlled by the internal volume of D-branes, these models naturally realize the idea of gauge coupling unification. They in addition permit to decouple the strength of gauge and gravitational interactions, even to the extent to implement the large extra dimension scenario [55, 56].

- 由于这种定域性，且四维规范耦合由 D 膜的内体积控制，这类模型自然实现了规范耦合统一的思想。此外，它们还可以解耦规范相互作用和引力相互作用的强度，甚至可以实现大额外维场景 [55, 56]。

- One can easily combine these models with additional ingredients that implement mechanisms for moduli stabilization at large volume, like background fluxes [57], in order to build more sophisticated models; see, e.g., [58].

- 可以很容易地为这类模型添加额外要素，实现大体积下模稳定的机制，例如背景流量 [57]，从而构建更复杂的模型；参见例如 [58]。

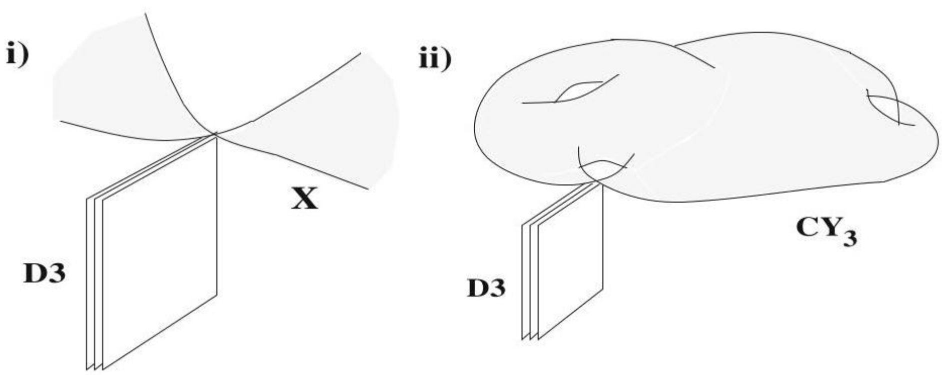


Fig. 7 Two-step procedure for building 4*d* models based on D3-branes at singularities. (Figure taken from [54])

图 7 基于奇点处 D3 膜构建 4d 模型的两步流程。(图引自 [54])

In the following we will discuss two classes of models that illustrate how these features are realized: D-branes at singularities and intersecting D7-branes. The latter can be thought of as a particular case of the F-theory constructions of section "F-Theory Model Building," which also incorporate these attractive features.

接下来我们将讨论两类模型，说明上述特征如何实现: 奇点处的 D 膜和相交 D7 膜。后者可以看作"F 理论模型构建"章节中 F 理论构造的特例，这类 F 理论构造也同样具备这些吸引人的特征。

D-Branes at Singularities

奇点处的 D 膜

Strictly speaking, D-branes at singularities do not correspond to large volume models that can be treated in the 10d supergravity regime. They are engineered in neighborhoods of the compact manifold X_6 that display a singular geometry, obtained for instance from collapsing some of its cycles. Nevertheless, one can analyze this system by directly quantizing open strings in a such a singular geometry, following the techniques initiated in [59,60], and then embed the resulting gauge sector into a fully fledged compactification, along the lines of the bottom-up philosophy described above.

严格来说，奇点处的 D 膜并不对应可在 10 维超引力 regime 下处理的大体积模型，它们构建在紧致流形 X_6 呈现奇异几何的邻域内，这类奇异几何例如可由流形的某些闭链坍缩得到。尽管如此，我们仍可遵循文献 [59,60] 开创的方法，直接对这类奇异几何中的开弦量子化，再按照上文所述自下而上的思路，将得到的规范场论 sector 嵌入完整的紧致化中。

The simplest example of this class of models is given by D3-branes at orbifold singularities. A stack of N D3-branes in this flat space yields a $4d\mathcal{N} = 4$ $U(N)$ gauge theory and therefore a non-chiral gauge sector. The non-chiral nature of the gauge sector persists if the D3-brane is placed at any smooth point of a compactification manifold X_6 , since the effect of curvature and background fluxes can at best lead to a massive deformation of 4 d$\mathcal{N} = 4$ SYM. The only way to obtain a chiral spectrum is to place the D3-brane on top of a singular geometry, like the one obtained by an orbifold quotient of the form $\mathbb{C}^3/\Gamma$ with fixed points. Let us for simplicity consider the cyclic orbifold group $\Gamma = \mathbb{Z}_k$ generated by an action on $\mathbb{C}^3$ of the form:

这类模型最简单的例子是轨形奇点处的 D3 膜。在平坦空间中，一堆 N 张 D3 膜给出 $4d\mathcal{N} = 4$ $U(N)$ 规范理论，即非手征规范 sector。只要 D3 膜放置在紧致化流形 X_6 的任意光滑点，非手征性就会保持，因为曲率和背景通量的作用最多只会给 4 d$\mathcal{N} = 4$ 超对称杨-米尔斯理论带来有质量形变。要得到手征谱，唯一的方式是将 D3 膜放置在奇异几何上方，例如带有不动点的 $\mathbb{C}^3/\Gamma$ 形式轨形商给出的几何。为简单起见，我们考虑由如下形式作用在 $\mathbb{C}^3$ 上生成的循环轨形群 $\Gamma = \mathbb{Z}_k$:

$$\left(z^1, z^2, z^3\right) \mapsto \left(e^{2\pi i \frac{a_1}{k}} z^1, e^{2\pi i \frac{a_2}{k}} z^2, e^{2\pi i \frac{a_3}{k}} z^3\right), \ a_i \in \mathbb{Z}. \tag{25}$$

In order for spinors to be well-defined in this background, one needs to require that $\sum_i a_i \in 2\mathbb{Z}$. Then one can embed Γ into $SU(4)$ as $\mathrm{diag}\left(e^{2\pi i \frac{b_0}{k}}, e^{2\pi i \frac{b_1}{k}}, e^{2\pi i \frac{b_2}{k}}, e^{2\pi i \frac{b_3}{k}}\right)$ with $b_i \in \mathbb{Z}$ and $\sum_i b_i = 0 \mod k$, such that $a_1 = b_2 + b_3, a_2 = b_3 + b_1$ and $a_3 = b_1 + b_2$, and quantize closed strings in this background [61]. To embed this singularity into a CY geometry, one must impose local $SU(3)$ holonomy, which amounts to $a_1 + a_2 + a_3 = 0 \mod k$. Then one can assume $b_0 = 0$ and $b_i = -a_i$.

为了让旋量在该背景下有良好定义，需要满足 $\sum_i a_i \in 2\mathbb{Z}$ 。随后可以将 Γ 作为 $\mathrm{diag}\left(e^{2\pi i\frac{b_0}{k}}, e^{2\pi i\frac{b_1}{k}}, e^{2\pi i\frac{b_2}{k}}\right.$ 、 $\left.e^{2\pi i\frac{b_3}{k}}\right)$ 嵌入 $SU(4)$ ，其中满足 $b_i \in \mathbb{Z}$ 和 $\sum_i b_i = 0 \mod k$ ，因此有 $a_1 = b_2 + b_3, a_2 = b_3 + b_1$ 和 $a_3 = b_1 + b_2$ ，再对该背景下的闭弦量子化 [61]。要将该奇点嵌入卡拉比-丘几何，必须要求局域 $SU(3)$ 和乐，这等价于 $a_1 + a_2 + a_3 = 0 \mod k$ 。进而可以假设 $b_0 = 0$ 和 $b_i = -a_i$ 。

Placing a stack of N D3-branes at the fixed point of (25) yields at gauge sector that is an orbifold projection of the initial 4 d$\mathcal{N} = 4U(N)$ gauge theory. The result depends on how the orbifold generator acts on the D3-brane Chan-Paton degrees of freedom, which is specified by an element of $U(N)$ of the form:

将一堆 N 张 D3 膜放置在 (25) 的不动点处，得到的规范 sector 是初始 4 d$\mathcal{N} = 4U(N)$ 规范理论的轨形投影。结果取决于轨形生成元如何作用在 D3 膜的陈-帕顿自由度上，这由 $U(N)$ 中如下形式的元素指定:

$$\gamma = \mathrm{diag}\left(\mathbb{1}_{N_0}, \omega\mathbb{1}_{N_1}, \dots, \omega^{k-1}\mathbb{1}_{N_{k-1}}\right), \ \omega = e^{\frac{2\pi i}{k}}, \tag{26}$$

with $\sum_{a=0}^{k-1} N_a = N$. The 4 d$\mathcal{N} = 4$ vector multiplet gets projected out to those Chan-Paton degrees of freedom λ invariant under the adjoint action $\lambda \mapsto \gamma\lambda\gamma^{-1}$, while for the three adjoint chiral multiplets Φ^i , only the modes invariant under $\lambda \mapsto e^{-2\pi i\frac{a_i}{k}}\gamma\lambda\gamma^{-1}$ survive. This results in the following spectrum:

带有 $\sum_{a=0}^{k-1} N_a = N$ 。4 d$\mathcal{N} = 4$ 向量多重态仅投影出在伴随作用 $\lambda \mapsto \gamma\lambda\gamma^{-1}$ 下不变的陈-帕顿自由度 λ ，而对于三个伴随手征多重态 Φ^i ，仅保留在 $\lambda \mapsto e^{-2\pi i\frac{a_i}{k}}\gamma\lambda\gamma^{-1}$ 下不变的模式。由此得到的能谱如下:

$$\text{Vector multiplet}: \prod_{a=0}^{k-1} U(N_a) \ , \tag{27}$$

$$\text{Chiral multiplets}: \sum_{a=0}^{k-1}\left[\left(\mathbf{N}_a, \overline{\mathbf{N}}_{a+a_1}\right) + \left(\mathbf{N}_a, \overline{\mathbf{N}}_{a+a_2}\right) + \left(\mathbf{N}_a, \overline{\mathbf{N}}_{a+a_3}\right)\right] ,$$

with a set of Yukawa couplings that arise from truncation of the parent $\mathcal{N} = 4$ superpotential $W = \mathrm{Tr}\left(\Phi^1\left[\Phi^2, \Phi^3\right]\right)$. Notice that this is a particular case of the general spectrum of Table 1, with the chiral index I_{ab} determined by the orbifold twists a_i , and without the presence of those representations that arise due to the orientifold projection. Indeed, while orientifold planes are a necessary ingredient of the global construction, a singularity can be located at a point $p \in X_6$ away from any O-plane. Then the orientifold projection simply requires that there is an identical singularity with similar D-brane content located at $\mathcal{R}p$.

并带有一组由母理论 $\mathcal{N}=4$ 超势 $W=\operatorname{Tr}\left(\Phi^1\left[\Phi^2,\Phi^3\right]\right)$ 截断得到的汤川耦合。注意这是表 1 通用能谱的一个特殊情形: 手征指标 I_{ab} 由轨形扭转 a_i 决定，且不包含 orientifold 投影所产生的那些表示。事实上，尽管 orientifold 平面是全局构造的必要组分，奇点也可以位于远离所有 O 平面的点 $p\in X_6$ 处。此时 orientifold 投影仅要求在 $\mathcal{R}p$ 处存在一个拥有相似 D 膜组分的相同奇点。

This chiral spectrum has a limited capacity of family replication, which occurs when two or more orbifold twists a_i are equal mod k . This sets an upper bound of three families and makes the orbifold $\mathbb{Z}_3$ with twist $\{a_i\}=(1,1,-2)$ particularly attractive [54, 62] . One may take $N_0=3, N_1=2$ and $N_2=1$, yielding a SM gauge group with hypercharge $U(1)_Y=U(1)_0/3+U(1)_1/2+U(1)_2$ and a partial SM chiral spectrum. The spectrum can then be completed by considering a stack of D7-branes going through the orbifold singularity and with a non-trivial action of the orbifold group on their Chan-Paton degrees of freedom [54].

这种手征能谱的代复制能力有限，当两个或更多轨形扭转 a_i 模 k 相等时代复制才会发生。这给出了三代的上限，并使得扭转为 $\{a_i\}=(1,1,-2)$ 的轨形 $\mathbb{Z}_3$ 格外有吸引力 [54, 62] 。我们可以取 $N_0=3, N_1=2$ 和 $N_2=1$ ，得到带有超荷 $U(1)_Y=U(1)_0/3+U(1)_1/2+U(1)_2$ 的标准模型规范群和部分标准模型手征能谱。随后可以通过引入一组穿过轨形奇点，且轨形群对其陈-帕顿自由度有非平凡作用的 D7 膜，补全完整能谱 [54]。

Besides the particularities of each model, there is a series of general features common to all of them that are worth mentioning:

除了每个模型自身的特殊性，所有模型都有一系列值得一提的共同特征:

- These models directly realize gauge coupling unification at the compactification scale because the coupling of all the gauge groups in (27) is given by the 4d dilaton. When introducing D7-branes, these may carry their own gauge group, but their gauge couplings are suppressed with respect to the local ones by the volume of the four-cycle wrapped by the D7-brane, and so from the viewpoint of the local model, they are treated as flavor branes. In general, the role of gauge groups coming from D7-branes can only be determined upon the global completion of the local model.

- 这些模型直接在紧致化尺度实现规范耦合统一，因为 (27) 中所有规范群的耦合都由四维伸缩子决定。引入 D7 膜时，D7 膜可以携带自身的规范群，但它们的规范耦合相较于局域模型的规范耦合会被 D7 膜所卷绕四周期的体积压低，因此对局域模型而言，D7 膜被当作味膜处理。一般来说，来自 D7 膜的规范群的作用只有在局域模型补全为全局构造后才能确定。

- There is always an anomaly-free massless $U(1)$ combination given by

- 始终存在一个无反常的无质量 $U(1)$ 组合，形式为

$$U(1)_{\text{diag}}=\sum_{a=0}^{k-1}\frac{U(1)_a}{N_a}, \tag{28}$$

which plays the role of hypercharge in the $\mathbb{C}^3/\mathbb{Z}_3$ model discussed above. Other $U(1)$ symmetries are typically anomalous and acquire a mass of the order of the string scale via a Green-Schwarz mechanism [63].

它在上述 $\mathbb{C}^3/\mathbb{Z}_3$ 模型中扮演超荷的角色。其他 $U(1)$ 对称性通常是反常的，会通过格林-施瓦茨机制获得弦尺度量级的质量 [63]。

- The gauge and chiral content of these local models can be encoded in a quiver diagram, in which each node represents a gauge group and a set of arrows connecting them the bi-fundamental representations. A D3-brane on a given node is dubbed fractional D3-brane, and the set of D3-branes that add up to the regular representation in (26) is identified as a bulk D3-brane that can be separated from the singularity and probe the bulk of X_6 . One can also extend the diagram to incorporate flavor D7-branes and their associated chiral content; see [64] for a short review.

- 这些局域模型的规范与手征物质内容可以用箭图表示，其中每个节点代表一个规范群，连接节点的一组箭头对应双基本表示。给定节点上的 D3 膜称为分数 D3 膜，而整体对应 (26) 中正则表示的一组 D3 膜，则被认定为可从奇点分离、用于探测 X_6 体空间的体 D3 膜。也可以扩展箭图来引入味 D7 膜及其对应的手征内容；简短综述见文献 [64]。

- When choosing (26), or more generally the D-branes in the quiver, one has to observe the local RR tadpole conditions. These form a subset of the whole set of tadpole conditions of the compactification which is only sensitive to fractional D3-brane and D7-brane charges. As usual satisfying these conditions implies that the non-abelian and mixed anomalies cancel [54].

- 选取 (26)，或更一般地选取箭图中的 D 膜时，必须满足局域 RR 蝌蚪条件。这类条件是紧致化全部蝌蚪条件的一个子集，仅与分数 D3 膜和 D7 膜的荷有关。和通常情况一样，满足这些条件意味着非阿贝尔反常和混合反常会消除 [54]。

Additionally, this simple orbifold setup can be generalized in a number of ways, which give rise to more and more sophisticated models, and which we briefly summarize in the following:

此外，这个简单的 orbifold 设置可以从多个方向推广，得到越来越复杂的模型，我们将在下文中简要总结:

- One may study more general orbifold groups Γ such as $\mathbb{Z}_k \times \mathbb{Z}_m$, or non-abelian subgroups of $SU(3)$ like for instance $\Gamma = \Delta_{27}$, which also features family triplication [54].

- 可以研究更一般的 orbifold 群 Γ ，例如 $\mathbb{Z}_k \times \mathbb{Z}_m$ ，或 $SU(3)$ 的非阿贝尔子群，比如 $\Gamma = \Delta_{27}$ ，这类结构也具有代三重化的特征 [54]。

- One may consider non-orbifold toric singularities such as conifold or del Pezzo singularities. In this case the gauge theory data are more efficiently encoded in a so-called dimer diagram [65], which is a tiling of T^2 . While these models provide more flexibility, certain features like the upper bound on three families remain generic [66].

- 可以考虑非 orbifold 的锥形奇点，例如锥奇点或德尔佩佐奇点。这种情况下，规范理论的数据可以更高效地编码在所谓的二聚体图中 [65]，二聚体图是 T^2 的一个平铺。这类模型虽然提供了更高的灵活性，但三代数的上界这类性质仍然是普适的 [66]。

- One may explore orientifolded singularities. In particular certain orientifolded del Pezzo singularities give rise to realistic SM spectra without the need of D7-branes [67], using the full spectrum of Table 1. Note that this result is in agreement with the general model building philosophy of section "D-Brane Model Building: Generalities," in the sense that one can build a realistic spectrum that avoids the necessity of additional D-brane sectors by cancelling all anomalies locally.

- 可以研究带 orientifold 投影的奇点。利用表 1 的完整谱，某些带 orientifold 投影的德尔佩佐奇点可以得到现实的标准模型谱，不需要引入 D7 膜 [67]。该结果符合“D 膜模型构建: 概述”一节的一般模型构建理念: 通过在局域消除所有反常，可以构建出现实谱，无需额外的 D 膜 sector。

Finally, it is worth mentioning the efforts to perform systematic embeddings of local models of D-branes at singularities into global compactifications, mostly using toric geometry techniques; see, e.g., [68-70].

最后，值得一提的是，目前已有不少工作利用锥形几何技术，将奇点处 D 膜局域模型系统地嵌入整体紧致化；例如见文献 [68-70]。

Intersecting D7-Branes

相交 D7 膜

Models of D-branes at singularities are a particular class of a broader set, which in the generic regime without collapsed cycles correspond to models of intersecting D7-branes. In this case the basic object is a D7-brane wrapping a holomorphic four-cycle $\Pi_4 \subset X_4$, threaded by a non-trivial worldvolume field strength $\mathcal{F}$ defined as in (12). The full spectrum between two D7-branes wrapping 4-cycles Π_4^a and Π_4^b must be computed in terms of Ext groups [71], but in order to determine the chiral index of Table 1, one may use the Riemann-Roch-Hirzebruch theorem to arrive at the expression [72]:

奇点处的 D 膜模型是更广泛集合中的一个特殊子类，在没有坍缩闭链的一般区域中，这类模型对应相交 D7 膜模型。在该情形下，基本对象是一张缠绕全纯四闭链 $\Pi_4 \subset X_4$ 的 D7 膜，缠绕体上带有如 (12) 定义的非平凡世界面场强 $\mathcal{F}$ 。两张分别缠绕四闭链 Π_4^a 和 Π_4^b 的 D7 膜之间的完整能谱必须用 Ext 群计算 [71]，但如果仅需要确定表 1 的手征指标，可以利用黎曼-罗赫-希策布鲁赫定理得到如下表达式 [72]:

$$I_{ab} = \int_{X_6} [\Pi_4^a] \wedge [\Pi_4^b] \wedge (c_1(F_a) - c_1(F_b)) . \tag{29}$$

Here $[\Pi_4^a]$ is the two-form Poincaré dual to the divisor class of Π_4^a , and $c_1(F_a)$ is the first Chern class of the quantized piece of the worldvolume flux $\mathcal{F}$ threading Π_4^a , viewed as an element of $H^2(X_6, \mathbb{Z})$. This formula extends to orientifold images by using that $\left(\Pi_4^{a'}, F_{a'}\right) = (\mathcal{R}\Pi_4^a, -\mathcal{R}F_a)$, and to the orientifold as $I_{aO} = 2\int_{X_6} [\Pi_4^a] \wedge [\Pi_{O7}] \wedge c_1(F_a)$. With these expressions at hand, the strategy to build models works quite similarly to the Type IIA case, and for instance one may reproduce the Pati-Salam sector of Table 6 by using three stacks of intersecting branes [39,40].

此处 $[\Pi_4^a]$ 是 Π_4^a 除子类的庞加莱对偶二形式，$c_1(F_a)$ 是穿过 Π_4^a 的世界面通量 $\mathcal{F}$ 量子化部分的第一陈类，被视作 $H^2(X_6,\mathbb{Z})$ 中的一个元素。该公式利用 $\left(\Pi_4^{a'}, F_{a'}\right) = (\mathcal{R}\Pi_4^a, -\mathcal{R}F_a)$ 推广到定向对称映像，利用 $I_{aO} = 2\int_{X_6}[\Pi_4^a] \wedge [\Pi_{O7}] \wedge c_1(F_a)$ 推广到定向对称本身。有了这些表达式，模型构建的思路与 IIA 型情形非常相似，例如可以通过三堆相交膜重现表 6 的帕蒂-萨拉姆 Sector[39,40]。

A natural arena for D7-brane model building is in the context of local models, following the general philosophy outlined above. The gauge group is now localized on a (fluxed) non-trivial four-cycle Π_4 which can be collapsed by moving in moduli space, and that may host chiral matter either via self-intersection or via the intersection with flavor D7-branes. Contracting such a four cycle leads to a singularity of the type described above with the wrapped D7-branes becoming fractional D3-branes [73]. While this shows that the two classes of Type IIB models under discussion are secretly similar to each other, in practice the model building possibilities are quite different. The reason is that the spectrum of BPS D-branes at the singular point and at volumes large compared to the string scale are quite different. In this sense both classes of models should be considered separately.

遵循上文概述的一般思路，局域模型是 D7 膜模型构建的自然应用场景。规范群定域在一个带通量的非平凡四闭链 Π_4 上，该四闭弦可通过在模空间中移动发生坍缩，并且可以通过自相交或者与味 D7 膜相交容纳手征物质。缩并这样一个四闭弦会产生前文所述类型的奇点，缠绕后的 D7 膜会变为分数 D3 膜 [73]。这说明本文讨论的两类 IIB 模型本质上彼此相似，但实际中模型构建的可能性差异很大，原因在于奇点处以及体积远大于弦标度处的 BPS D 膜能谱截然不同。从这个角度看，两类模型应当分开讨论。

Still, they have a number of similarities. If the volume of the contractible four-cycle Π_4 is large enough, one may achieve an approximate gauge coupling unification. This is because the gauge coupling constant associated with a D7-brane gauge group is set by its volume and its worldvolume flux, and in the regime of dilute flux densities, the former is the dominant contribution. This suggests GUT model building as an obvious model building option, and in particular SU(5) GUTs, with a 10 representation coming from intersection with an O7-plane [72]. These models suffer from problems similar to the ones mentioned in section "Yukawa Couplings," in the sense that the massive $U(1)$ symmetry within $U(5)$ forbids the generation of the top Yukawa coupling $\mathbf{10105}_H$ at the perturbative level. This motivates applying the same model building philosophy but in the more general context of F-theory GUTs, to be discussed in section "F-Theory Model Building." Given the similarities between F-theory and D7-brane GUTs, we will refrain from further discussing the latter, and refer the reader to [62,72] for details.

尽管如此，二者仍存在诸多相似点。如果可缩并四闭链 Π_4 的体积足够大，就可以实现近似规范耦合统一。这是因为 D7 膜规范群对应的规范耦合常数由其体积和世界面通量共同决定，在低密度通量区域，体积是主导贡献。这提示大统一理论 (GUT) 模型构建是自然的选择，尤其是 SU(5) 大统一，其 10 维表示来自与 O7 平面的相交 [72]。这些模型存在的问题与“汤川耦合”一节提到的问题类似: $U(5)$ 框架下的整体 $U(1)$ 对称性禁止顶夸克汤川耦合 $\mathbf{10105}_H$ 在微扰阶产生。这推动人们将相同的模型构建思路应用到更一般的 F 理论大统一框架中，我们将在“F 理论模型构建”一节讨论。由于 F 理论和 D7 膜大统一存在诸多相似性，本文不再对后者展开讨论，感兴趣的读者可以查阅文献 [62,72] 了解细节。

Rational Conformal Field Theories

有理共形场论

A conformal field theory (CFT) is a field theory with invariance under the conformal group. Here we are only interested in two-dimensional CFTs living on the worldsheet of strings. The properties (such as spectra and correlation functions) of such a theory can be described in terms of the algebra of conformal field theory, the Virasoro algebra. This algebra is characterized by a number c called the central charge. For $c < 1$ the representation theory of the Virasoro algebra is well-known [74]. It has a discrete series of unitary representations for certain rational values of c , accumulating at $c = 1$. For each c in the series, there is a finite number of representations, characterized by a conformal weight denoted as h . Conformal field theories with a finite number of representations are called rational conformal field theories, or RCFT for short.

共形场论 (CFT) 是在共形群下保持不变的场论。本文我们仅关注存在于弦世界面上的二维共形场论。这类理论的性质 (例如谱和关联函数) 可以通过共形场论的代数，即 Virasoro 代数来描述。该代数由一个称为中心荷的量 c 表征。对于 $c < 1$ ，Virasoro 代数的表示理论已经被充分研究 [74]。当 c 取特定有理值时，它存在一系列离散的幺正表示，这些表示在 $c = 1$ 处集聚。对于该序列中的每个 c ，都存在有限个表示，由共形权重 h 表征。拥有有限个表示的共形场论被称为有理共形场论，简称 RCFT。

Conformal field theory encompasses all worldsheet descriptions of perturbative string theory, including geometric orbifolds and orientifolds, like the ones discussed in the previous section. But in those cases the geometric language provides the more powerful description. Using RCFTs will allow us to go into uncharted territory not easily accessible geometrically.

共形场论涵盖了微扰弦论所有的世界面描述，包括上一节讨论过的几何轨道形和定向形。但在这些场景中，几何语言能提供更有力的描述。使用 RCFT 可以让我们进入几何方法难以触及的未知领域。

In addition to the Virasoro algebra, generated by currents of spin 2, other algebras may be present. They may be generated by currents of spin 1 (affine Lie algebras, often called Kac-Moody algebras), higher spin (called W algebras), spin-$\frac{1}{2}$ (free fermions) and spin-$\frac{3}{2}$ (superconformal algebras). As a general rule, the extra symmetry makes representations larger, and reduces their number. In particular, the number of representations may become finite, producing an RCFT. The set of generators, acting on either the right- or left-moving modes of the theory, is called the chiral algebra.

除了由自旋 2 流生成的 Virasoro 代数外，还可能存在其他代数。它们可以由自旋 1 流 (仿射李代数，常称为 Kac-Moody 代数)、更高自旋流 (称为 W 代数)、自旋 $\frac{1}{2}$ 流 (自由费米子) 和自旋 $\frac{3}{2}$ 流 (超共形代数) 生成。一般来说，额外对称性会让表示更大，同时减少表示的数量。尤其值得一提的是，表示的数量可以变为有限，从而得到 RCFT。作用在理论右行模或左行模上的生成元集合被称为手征代数。

Gepner Models

格普纳模型

Compactified closed superstring theories are often described geometrically, as strings propagating in a space with D flat dimensions, and $10-D$ dimensions rolled up on a torus, an orbifold or a Calabi-Yau manifold. But there is an alternative description in terms of a suitable CFT on the worldsheet.

紧化闭超弦理论通常采用几何描述，即弦在拥有 D 个平坦维度、$10-D$ 个卷在环面、轨形或卡拉比-丘流形上的维度的空间中传播。但也可以利用世界面上合适的共形场论给出替代描述。

To characterize the compactified sector of a superstring with four uncompactified dimensions, we need superconformal field theories with a total central charge of nine, and two supersymmetries. This is because the uncompactified theory is defined by worldsheet fields X^I and $\Psi^I, I=1,\dots,6$. The free worldsheet theory has $\mathcal{N}=2$ supersymmetry, and its conformal field theory has a total central charge of $6+3=9$ (each worldsheet boson contributes 1, each fermion $\frac{1}{2}$). The compactified theory has to mimic these properties to preserve the consistency of the theory. In short, we need a $c=9, \mathcal{N}=2$ superconformal field theory.

要刻画拥有四个未紧化维度的超弦的紧化区域，我们需要总中心荷为 9、且带有两个超对称的超共形场论。这是因为未紧化理论由世界面场 X^I 和 $\Psi^I, I=1,\dots,6$ 定义。自由世界面理论拥有 $\mathcal{N}=2$ 超对称，其共形场论总中心荷为 $6+3=9$ (每个世界面玻色子贡献 1，每个费米子贡献 $\frac{1}{2}$)。紧化理论必须满足这些性质才能保持理论自洽。简而言之，我们需要一个 $c=9, \mathcal{N}=2$ 超共形场论。

The $\mathcal{N}=2$ superconformal field theories also have a discrete series, this time accumulating at $c=3$. The values of c are

$\mathcal{N}=2$ 超共形场论同样存在离散序列，该序列在 $c=3$ 处累积。c 的取值为

$$c=\frac{3k}{k+2} \tag{30}$$

These $\mathcal{N}=2$ minimal models are superconformal RCFTs, but they do not have the required central charge of 9 . This can be solved by ”tensoring” several copies of them, in such a way that the sum of their central charges is 9 . There are 168 solutions $(k_1,\dots,k_n)$ to the equations:

这些 $\mathcal{N}=2$ 极小模型都是超共形有理共形场论，但它们不满足要求的中心荷 9。可以通过“张量积”多份该模型解决这个问题，使得它们的中心荷之和为 9。方程共有 168 个解 $(k_1,\dots,k_n)$:

$$\sum_i \frac{3k_i}{k_i+2}=9 \tag{31}$$

One needs four to nine copies, for example, $(3,8,8,8)$ or $(1,1,1,1,1,1,1,1,1)$. This method was first used by D. Gepner [75] to construct compactified heterotic strings, and hence these tensor products are often referred to as Gepner models. Here we are using them to describe the compactified sector of a Type II superstring.

需要用到 4 到 9 个拷贝，例如 $(3,8,8,8)$ 或 $(1,1,1,1,1,1,1,1,1)$ 。该方法最早由 D. 格普纳 [75] 用于构造紧化杂化弦，因此这类张量积通常被称为格普纳模型。本文中我们用它来刻画 II 型超弦的紧化区域。

Gepner models are by no means the only way to construct superconformal RCFTs with $c = 9$, but they are the only ones that are both non-trivial and mathematically under control, so that relevant quantities are computable. Therefore they present an excellent theoretical laboratory for investigating both closed and open string models.

格普纳模型绝非构造拥有 $c = 9$ 的超共形有理共形场论的唯一方法，但它是唯一一类既非平凡又在数学上可控，相关物理量都可以计算的构造。因此格普纳模型是研究闭弦和开弦模型的极佳理论实验室。

Modular Invariant Partition Functions

模不变配分函数

After choosing an RCFT, a second important choice to be made is that of the modular invariant partition function (MIPF), which enters the discussion when the worldsheet diagram is a torus. In closed string theories, this diagram encodes the closed string spectrum, in terms of a partition function:

选定有理共形场论 (RCFT) 后，第二个需要确定的重要选择是模不变配分函数 (MIPF)，当世界面图为环面时就会涉及到这个概念。在闭弦理论中，该图通过配分函数编码了闭弦的能谱:

$$\int \mathcal{D}X e^{-S_E(X)} = \mathrm{Tr}\, e^{2\pi i\tau(L_0-1)} e^{-2\pi i\bar{\tau}(\bar{L}_0-1)}, \tag{32}$$

where the left-hand side denotes - symbolically - the path integral. The complex parameter τ denotes the modular parameter of the torus, which describes its shape. By expanding in powers of

其中左侧象征性地表示路径积分。复参数 τ 表示环面的模参数，描述环面的形状。对

$$q = e^{2\pi i\tau} \text{ and } \bar{q} = e^{-2\pi i\bar{\tau}} \tag{33}$$

one can read off the multiplicities of the closed string spectrum. Only states with the same power of q and $\bar{q}$ are physical.

做幂次展开后，就可以读出闭弦能谱的简并度。只有 q 和 $\bar{q}$ 幂次相同的态才是物理态。

The right-hand side of (32) can be expanded in terms of characters of the (extended) conformal field theory:

(32) 的右侧可以按 (扩展) 共形场论的特征标展开:

$$\chi_i = \mathrm{Tr}\,|_i e^{2\pi i\tau(L_0-1)}, \tag{34}$$

where the trace is over all states in the representation built on ground state i by the action of the Virasoro generators L_n and the generators of some extension of the Virasoro algebra. The characters have the following expansion:

其中迹是对由 Virasoro 生成元 L_n 和 Virasoro 代数某种扩展的生成元作用在基态 i 上构造出的表示中所有态取的。特征标满足如下展开式:

$$\chi_i(\tau) = q^{h_i - c/24} \sum_n d_n q^n, \tag{35}$$

where d_n are nonnegative integers.

其中 d_n 为非负整数。

The number of such representations will in general be infinite, but if the extension of the symmetry algebra is large enough we have an RCFT, and then the number is finite. The character label 0 corresponds to the vacuum representation. It contains the vacuum and all states obtained from it by the action of the Virasoro generators and all generators of the extended algebra (if any), modulo states of zero norm (null states).

一般来说这类表示的数量是无穷多的，但如果对称代数的扩展足够大，我们就得到一个有理共形场论，此时表示的数量是有限的。特征标标号 0 对应真空表示，它包含真空，以及 Virasoro 生成元和所有扩展代数生成元 (如果有的话) 作用在真空上得到的所有态，模去零范态 (零态)。

The torus partition function has the following character expansion:

环面配分函数的特征标展开形式如下:

$$\mathrm{Tr}\, e^{2\pi i\tau(L_0-1)} e^{-2\pi i\bar{\tau}(\bar{L}_0-1)} = \sum_{ij} Z_{ij} \chi_i(\tau) \overline{\chi}_j(\overline{\tau}). \tag{36}$$

Here the coefficients Z_{ij} must be nonnegative integers, subject to the constraint of modular invariance. This constraint is a consequence of the fact that there are infinitely many parametrizations of the torus that must all give the same result. These reparametrizations are generated by discrete transformations of the parameter τ :

此处系数 Z_{ij} 必须是非负整数，且满足模不变性约束。该约束源于一个事实: 环面存在无穷多种参数化方式，它们必须给出相同的结果。这些重参数化由参数 τ 的离散变换生成:

$$T : \tau \to \tau + 1 \tag{37}$$

$$S : \tau \to -\frac{1}{\tau}. \tag{38}$$

These transformations are represented as matrices S and T on the set of characters χ_i , and the conditions for modular invariance are therefore

这些变换在特征标集合 χ_i 上表示为矩阵 S 和 T ，因此模不变性的条件为

$$[S, Z] = [T, Z] = 0. \tag{39}$$

Furthermore Z_{00} , the multiplicity of the vacuum state, must be equal to 1 . In closed string theory, this has the consequence that there is precisely one graviton in the spectrum.

此外，真空态的简并度 Z_{00} 必须等于 1。在闭弦理论中，这一条件保证能谱中恰好存在一个引力子。

There has been a lot of work on finding solutions to the conditions for Z_{ij} , but this has been completed only for a few extended algebras. Notable examples are the unextended Virasoro algebra with $c < 1$ as discussed above, and the $SU(2)$ Kac-Moody algebra [76]. Furthermore there are known classes of general solutions that are valid for any RCFT: the charge conjugation-invariant $Z_{ij} = C_{ij}$, the diagonal invariant $Z_{ij} = \delta_{ij}$, conformal embeddings [77,78], and simple current invariants [79]. Here C_{ij} is a bijection that corresponds to charge conjugation in the worldsheet theory, acting on the ground states. It is known that the diagonal invariant is always a solution to the conditions of modular invariance at one loop, but does not always define a consistent CFT [80]. On the other hand, the charge conjugation invariant always defines a consistent CFT, and should be viewed as the canonical definition of the theory. We refer to this case as ”C-diagonal.”

人们已经围绕 Z_{ij} 条件的解做了大量工作，但仅对少数几个扩展代数完全解决了这个问题。值得注意的例子包括前文讨论的中心荷为 $c < 1$ 的非扩展 Virasoro 代数，以及 $SU(2)$ Kac-Moody 代数 [76]。此外，目前已经找到了适用于任意有理共形场论的几类通解: 电荷共轭不变解 $Z_{ij} = C_{ij}$ 、对角不变解 $Z_{ij} = \delta_{ij}$ 、共形嵌入 [77,78] 以及单流不变解 [79]。此处 C_{ij} 是对应世界面理论中电荷共轭的双射，作用在基态上。已知对角不变解总是单圈模不变性条件的解，但并不总能定义一个自洽的共形场论 [80]。另一方面，电荷共轭不变解总能定义一个自洽的共形场论，应当被视为该理论的标准定义。我们将这种情况称为“C 对角”。

Fusion Rules and Simple Currents

融合规则与简单流

The fusion rules of an RCFT indicate how many couplings exist when two representations $[i]$ and $[j]$ are combined. It can be formally written as

有理共形场论 (RCFT) 的融合规则给出了两个表示 $[i]$ 和 $[j]$ 组合时存在多少种耦合，形式上可写为

$$[i] \times [j] = \sum_k N_{ij}^k [k] \tag{40}$$

where $[i]$ denotes an (extended) CFT representation. A simple current [79, 81] is a special representation J with the particular feature that just one term (labeled Ji) exists on the right-hand side:

其中 $[i]$ 表示一个 (扩张后的) 共形场论表示。简单流 [79, 81] 是特殊表示 J ，其特点是右侧仅存在一项 (标记为 Ji):

$$[J] \times [i] = [Ji] \tag{41}$$

The set of representations generated by the action of J on $[i]$ is called the orbit of $[i]$.

由 J 作用在 $[i]$ 上生成的表示集合称为 $[i]$ 的轨道。

The reason simple currents enter the story is that they allow us to construct a large number of modular invariant partition functions. This works roughly as follows. The action of the simple currents on themselves defines an abelian discrete group. Take any subgroup of that discrete group. Now take a set of generators of that subgroup. On that basis of generators, one defines a matrix X . This is a matrix of rational numbers [82] computed from a simple equation, which occasionally has no solution, but generally has a number of solutions that grows exponentially with the size of the basis. We will omit the details here, but the main point is that for any subgroup of the simple current group, one has many modular invariants.

引入简单流的原因是它们可以用来构造大量模不变配分函数，大致思路如下: 简单流自身的作用定义了一个阿贝尔离散群，任取该离散群的一个子群，再取这个子群的一组生成元，在这组生成元的基础上可以定义矩阵 X 。这是一个有理矩阵 [82]，由一个简单方程计算得到，该方程偶尔无解，但一般来说解的数量随生成元基底的大小指数增长。我们在此省略细节，核心结论是: 对简单流群的任意子群，都可以得到大量模不变量。

The simple current group of $\mathcal{N} = 2$ minimal model is fairly large. For k odd it is $\mathbb{Z}_{4k}$ and for k even it is $\mathbb{Z}_2 \times \mathbb{Z}_{2k}$. But much more importantly, in a tensor product, one gets a product of all these groups. For example, the combination (3, 8, 8, 8) yields $\mathbb{Z}_{12} \times (\mathbb{Z}_2)^4 \times (\mathbb{Z}_{16})^4$. These discrete groups have a huge number of subgroups, and hence for every Gepner model, we get a huge number of MIPFs. This boosts the number of available Gepner models from 168 (for just the C-diagonal MIPF) to 5403.

$\mathcal{N} = 2$ 极小模型的简单流群相当大: 当 k 为奇数时，它是 $\mathbb{Z}_{4k}$ ；当 k 为偶数时，它是 $\mathbb{Z}_2 \times \mathbb{Z}_{2k}$ 。更重要的是，张量积的简单流群是所有因子群的直积，例如组合 (3, 8, 8, 8) 给出 $\mathbb{Z}_{12} \times (\mathbb{Z}_2)^4 \times (\mathbb{Z}_{16})^4$ 。这些离散群存在极多子群，因此对每个格普纳模型，我们都能得到极多模不变部分配分函数 (MIPF)，这将可用格普纳模型的数量从 168 个 (仅 C 对角 MIPF 的情况) 提升到了 5403 个。

Open String CFT

开弦共形场论

To use RCFTs for open string model building, one needs a description of ends of open strings. In a worldsheet description, they sweep out worldsheets with boundaries. The most general worldsheet for oriented closed strings is a Riemann surface of arbitrary genus g , which is a torus with g handles. One can attach tubes to act as external closed strings. To get all open string diagrams, one can make holes in those surfaces with the topology of a circle. Finally, one can add strips to the edges of the holes to act as external open strings.

要将有理共形场论用于开弦模型构建，需要对开弦的端点进行描述。在世界面描述中，开弦端点扫出的世界面带边界。定向闭弦最一般的世界面是任意亏格 g 的黎曼曲面，也就是带有 g 个柄的环面。我们可以接上管来表示外闭弦。要得到所有开弦图，我们可以在这类曲面上挖出拓扑为圆周的孔。最终，我们可以在孔的边缘接上带条来表示外开弦。

In section ”The Need for O-Planes” we have seen that O-planes are needed in order to build consistent open string models (at least in the supersymmetric case). Just as D-branes are described by means of boundaries of the surface, O-planes are described by means of another topological feature, a crosscap. A crosscap is added to a surface by making a hole, as above, but identifying the opposite points of the boundary circle to each other in an orientation-reversing way. Hence an ant crawling on one side of the circle finds itself on the other side after crossing the crosscap. One can add more than one crosscap, but not all resulting surfaces are topologically distinct. A sphere with two holes is topologically an annulus; a sphere with one hole and a crosscap is equivalent to a Moebius strip, and a sphere with two crosscaps is a Klein bottle.

在“O 平面的必要性”一节中我们已经知道，构建自洽的开弦模型 (至少在超对称情形下) 需要 O 平面。正如 D 膜用曲面的边界描述一样，O 平面由另一种拓扑特征——交叉帽描述。给曲面添加交叉帽的操作和上文一样先挖出一个孔，再将边界圆的对径点以反转定向的方式等同起来。因此，爬过交叉帽的蚂蚁会从圆周的一侧去到另一侧。我们可以添加不止一个交叉帽，但并非所有得到的曲面都是拓扑不同的。带两个孔的球面拓扑上是环带；带一个孔和一个交叉帽的球面等价于莫比乌斯带，带两个交叉帽的球面则是克莱因瓶。

Now we have to determine the behavior of the CFT near the edges of the surface or in the presence of a crosscap

现在我们需要确定共形场论在曲面边缘附近，或是存在交叉帽时的行为

Boundary and Crosscap States

边界态与交叉帽态

Any surface with boundaries or crosscaps has a double cover which only has handles, and on which one defines a closed, oriented conformal field theory. This CFT is the starting point for constructions of open (and unoriented) strings, which were referred to as ”open descendants” of the closed string theories in [83]. The presence of boundaries and crosscaps is described by boundary and crosscap ”state,” which are not really states themselves, but in fact non-normalizable linear combinations of states in the closed string Hilbert space.

任何带边界或交叉帽的曲面都存在仅含亏格的双叶覆叠，我们可在其上定义闭合、定向的共形场论。该 CFT 是构建开弦 (以及非定向弦) 的起点，这类开弦在文献 [83] 中被称为闭弦理论的“开后代”。边界与交叉帽的存在由边界和交叉帽“态”描述，它们本身并不是真正的态，实际上是闭弦希尔伯特空间中不可归一化的态线性组合。

Here we will assume that the entire chiral algebra remains unbroken at the boundary or by a crosscap, ignoring the interesting possibility of breaking part of the closed string symmetries. The condition that a symmetry is not broken by a boundary or a crosscap is

此处我们假设整个手征代数在边界或交叉 cap 处保持不破缺，暂不讨论闭弦部分对称性破缺这一重要情形。对称性不被边界或交叉 cap 破缺的条件为

$$\left(W_n + (-1)^{h_W}\widetilde{W_n}\right)\left|B\right\rangle = 0,\ \left(W_n + (-1)^{h_W+n}\widetilde{W_n}\right)\left|C\right\rangle = 0, \tag{42}$$

where W_n is a mode of a chiral current, $\widetilde{W_n}$ a mode of an anti-chiral current and h_W its conformal weight,

$|B\rangle$ is a boundary state and $|C\rangle$ is a crosscap state. A basis for the solutions to these conditions is formed by the Ishibashi states [84]:

其中 W_n 是手征流的模，$\widetilde{W}_n$ 是反手征流的模，h_W 是其共形权重，$|B\rangle$ 是边界态，$|C\rangle$ 是交叉帽态。满足这些条件的解的基由石桥态给出 [84]:

$$|B_i\rangle = \sum_I |I\rangle_i \otimes U_B |I\rangle_{i^c}, \ |C_i\rangle = \sum_I |I\rangle_i \otimes U_C |I\rangle_{i^c}. \tag{43}$$

Here the i labels a representation of the chiral algebra and i^c its charge conjugate. The sum is over all states in the representation, and U_B and U_C are operators satisfying

此处 i 标记手征代数的一个表示，i^c 是其电荷共轭表示。求和遍历表示中的所有态，且 U_B 和 U_C 是满足下式的算符

$$\widetilde{W}_n U_B = (-1)^{h_W} U_B \widetilde{W}_n, \ \widetilde{W}_n U_C = (-1)^{h_W+n} U_C \widetilde{W}_n. \tag{44}$$

Any boundary state must be a linear combination of these Ishibashi states, i.e.,

任意边界态都必须是这些石桥态的线性组合，即

$$|B_a\rangle = \sum_i B_{ia} |B_i\rangle, \ \Big|C\rangle = \sum_i \Gamma_i \Big| C_i\rangle. \tag{45}$$

It turns out that in general one can allow for several boundary states, labeled by a boundary label a , but for only one crosscap state for a given theory. For a given MIPF Z_{ij} , more than one crosscap state may exist, but one cannot mix them.

可以证明，一般而言允许存在多个边界态，由边界标记 a 区分，但一个给定理论仅对应一个交叉帽态。对于给定的 MIPF Z_{ij} ，可能存在多个交叉帽态，但它们不能混合。

A choice of a set of boundary labels a , and a set of coefficients B_{ai} and Γ_i form part of the data that define an open string CFT. Although more is required to specify all correlation functions on arbitrary surfaces, this information is sufficient to compute the one-loop diagrams without external lines that contribute to the open and closed string partition functions. The relevant string diagrams are those with vanishing Euler number. From these diagrams we can compute the spectrum of the theory.

一组边界标记 a ，加上一组系数 B_{ai} 和 Γ_i ，构成了定义开弦 CFT 的部分数据。尽管要指定任意曲面上的所有关联函数还需要更多信息，但这些信息足以计算对开弦和闭弦配分函数有贡献的无外线单圈图。相关的弦图是欧拉示性数为零的弦图。我们可以从这些图中计算出该理论的能谱。

Orientifold Partition Functions

定向轨形配分函数

In the presence of boundaries and crosscaps, there are four topologically distinct surfaces with vanishing Euler number: the torus, the Klein bottle, the annulus, and the Möbius strip. These contributions can be expanded in (bi)linears of characters:

当存在边界和交叉帽时，共有四个拓扑不同、欧拉数为零的曲面: 环面、克莱因瓶、环带和莫比乌斯带。这些贡献可以按特征标的 (双) 线性展开:

$$\mathcal{T} = \sum_{ij} Z_{ij} \chi_i(\tau) \chi_j^*(\tau), \ \mathcal{K} = \sum_i K^i \chi_i(2it), \tag{46}$$

$$\mathcal{A} = \sum_{ab} N_a N_b A^i_{ab} \chi_i(it/2), \ \mathcal{M} = \sum_a N_a M^i_a e^{-i\pi(h_i - c/24)} \chi_i\left(\frac{1}{2}(1+it)\right)(\tag{47}$$

Here τ is the modular parameter of the torus, as before, and $t = \operatorname{Im}\tau$. As discussed in section ”Modular Invariant Partition Functions,” the torus defines the oriented closed string partition function. Likewise, the sum of the torus and the Klein bottle defines the unoriented closed string partition function. The annulus is an open string loop, and defines, together with the Möbius strip, the open string partition function. The Klein bottle amplitude and the Möbius strip act as an orientifold projection on the closed and open string spectrum, respectively.

此处 τ 和前文一样，是环面的模参数，而 $t = \operatorname{Im}\tau$ 。正如“模不变配分函数”一节所讨论的，环面对应了定向闭弦的配分函数。类似地，环面与克莱因瓶的和对应了无定向闭弦的配分函数。环带是开弦圈，它和莫比乌斯带共同定义了开弦配分函数。克莱因瓶振幅和莫比乌斯带分别对闭弦谱和开弦谱起到定向轨形投影的作用。

Channel Transformations

道变换

The diagrams that describe the spectrum are computed in the transverse channel, in which closed strings propagate between two boundaries, a boundary and a crosscap, or two crosscaps. In Fig. 8 this is illustrated for the simplest case: a diagram of closed string propagation from boundary a to boundary b is transformed to a diagram where open strings with endpoints a and b propagate in a closed loop. This is called the direct channel. By the rules of string perturbation theory, this is the same diagram, but with two different parametrizations. To transform from one parametrization to another, one has to interchange the worldsheet space and time directions. This can be done by means of the transformation $\tau \to -\frac{1}{\tau}$, which acts on the characters as a matrix S . The analogous figure for non-orientable surfaces is harder to draw, but the direct channel for closed strings propagating between a boundary and a crosscap is a Moebius strip, and the direct channel for propagation between two crosscaps is a Klein bottle. The required transformations are also different. Here we will just give the result. The details and many references may be found in [16].

描述谱的图在横向道中计算，横向道中闭弦在两个边界、一个边界和一个交叉帽，或两个交叉帽之间传播。图 8 演示了最简单的情况: 从边界 a 传播到边界 b 的闭弦图，会变换为端点为 a 和 b 的开弦沿闭合回路传播的图。这被称为直接道。根据弦微扰论的规则，这是同一个图，只是有两种不同的参数化方式。要从一种参数化变换到另一种，需要交换世界面的空间方向和时间方向。这可以通过变换 $\tau \to -\frac{1}{\tau}$ 完成，该变换作为矩阵 S 作用在特征标上。不可定向曲面的对应图形更难绘制，但闭弦在边界和交叉帽之间传播的直接道是莫比乌斯带, 闭弦在两个交叉帽之间传播的直接道是克莱因瓶。所需的变换也不同，这里我们仅给出结果，细节和更多参考文献可在文献 [16] 中找到。

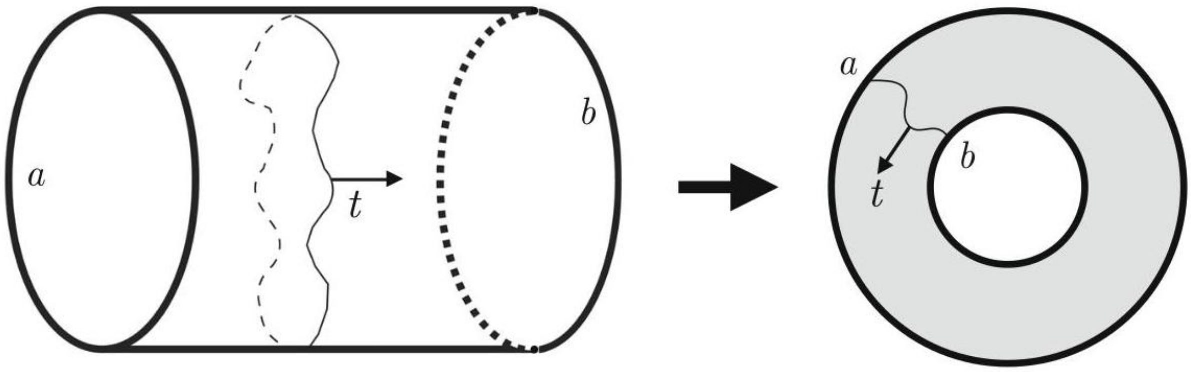


Fig. 8 Channel transformation

图 8 道变换

The three basic transverse channel amplitudes are obtained by sandwiching the closed string propagators between boundary and crosscap states:

三种基本的横向道振幅是通过将闭弦传播子夹在边界态与交叉帽态之间得到的:

$$\text{Transverse Annulus: } N_a N_b \left\langle B_a^c \right| e^{i\tau H} \left| B_b \right\rangle , \tag{48}$$

$$\text{Transverse Moebius strip: } N_a \left[\left\langle B_a^c \right| e^{i\tau H} \left| C \right\rangle + \left\langle C^c \right| e^{i\tau H} \left| B_a \right\rangle \right] , \tag{49}$$

$$\text{Transverse Klein bottle: } N_a \left\langle C^c \right| e^{i\tau H} \left| C \right\rangle .$$

(50)

Here H is the closed string Hamiltonian: $H = 2\pi \left(L_0 + \tilde{L}_0 - c/12 \right)$, and τ is a real number representing the length of the cylinder. The subscript " c " indicates that a CPT conjugate state is to be used. The integers N_a are the Chan-Paton multiplicities. One can express these amplitudes in terms of characters of the representation i . By means of a transformation of the parameter τ , one can then compute the corresponding amplitudes in the direct channel (the open and closed string loop channels). In the case of the Klein bottle and the annulus, this transformation acts on the characters as the modular transformation matrix S , whereas in the case of the Moebius strip, one uses the matrix $P = \sqrt{T} S T^2 S \sqrt{T}$, with $\sqrt{T}$ defined as $\exp\{i\pi (L_0 - c/24)\}$. Then one arrives at the following expressions:

此处 H 是闭弦哈密顿量: $H = 2\pi\left(L_0 + \tilde{L}_0 - c/12\right)$，$\tau$ 是代表圆柱长度的实数。下标“c”表示需要使用 CPT 共轭态。整数 N_a 是陈-帕顿多重度。我们可以将这些振幅用表示 i 的特征标表示。通过对参数 τ 做变换，就可以计算出直接道 (开弦和闭弦圈道) 中对应的振幅。对于克莱因瓶和环面，该变换以模变换矩阵 S 的形式作用在特征标上，而对于莫比乌斯带，我们使用矩阵 $P = \sqrt{T}ST^2S\sqrt{T}$，其中 $\sqrt{T}$ 定义为 $\exp\{i\pi(L_0 - c/24)\}$。由此我们得到了下述表达式:

$$\text{Direct Annulus:}\quad \frac{1}{2}N_aN_bA^i{}_{ab}\chi_i\left(\frac{1}{2}\tau\right), \tag{51}$$

$$\text{Direct Moebius strip:}\quad \frac{1}{2}N_aM_a^i\hat{\chi}_i\left(\frac{1}{2}+\frac{1}{2}\tau\right), \tag{52}$$

$$\text{Direct Klein bottle:}\quad \frac{1}{2}K^i\chi_i(2\tau). \tag{53}$$

Here $\hat{\chi}_i \equiv T^{-\frac{1}{2}}\chi_i$, and the parameter τ is purely imaginary. The coefficients are

此处 $\hat{\chi}_i \equiv T^{-\frac{1}{2}}\chi_i$，参数 τ 是纯虚数。系数为

$$A^i{}_{ab} = \sum_m S_m^i B_{ma}B_{mb}, \tag{54}$$

$$M_a^i = \sum_m P_m^i B_{ma}\Gamma_m, \tag{55}$$

$$K^i = \sum_m S_m^i \Gamma_m\Gamma_m \tag{56}$$

Integrality Conditions

整性条件

To interpret these expressions in terms of state counting, it is clearly important that all the relevant coefficients be nonnegative integers. Indeed, in all cases discussed here, Z_{ij} and A_{ab}^i are explicitly nonnegative integers, and K^i and M_a^i are integers. But that is not sufficient, because the actual state multiplicities are sums and differences of these numbers. Note first of all that the argument of the Klein bottle term,2it, coincides precisely with the terms in the expansion of $\chi_i(\tau)\,\chi_j^*(\tau)$ for the physical states: those with the same powers of q and $\bar{q}$. Hence this term may alter the multiplicity of the physical states. It is the total multiplicity that must be a nonnegative integer. The vacuum representation, $i = 0$, has $Z_{00} = 1$ and $K_0 = 1$. As stated before, this representation produces the graviton, which should have multiplicity 1. Hence we find that we must divide the entire partition function by 2 to get the right multiplicity. To get correct multiplicities for all the other closed string states, we need

为了从态计数的角度解释这些表达式，所有相关系数都必须是非负整数，这一点显然至关重要。实际上，在本文讨论的所有情况中，Z_{ij} 和 A^i_{ab} 都是明确的非负整数，K^i 和 M^i_a 都是整数。但这并不充分，因为实际的态简并度是这些数的和与差。首先注意，克莱因瓶项的宗量 $2it$ 与物理态对应的 $\chi_i(\tau)\chi^*_j(\tau)$ 展开中的项完全一致: 这些项的 q 和 $\bar{q}$ 幂次都相同。因此该项会改变物理态的简并度。总简并度必须是非负整数。真空表示 $i=0$ 满足 $Z_{00}=1$ 和 $K_0=1$ 。如前所述，该表示给出引力子，其简并度应为 1。因此我们发现必须将整个配分函数除以 2 才能得到正确的简并度。为了让所有其他闭弦态的简并度都正确，我们需要

$$K^i = Z_{ii} \bmod 2 \text{ and } |K_i| \leq Z_{ii},$$

which is indeed satisfied in all known case. Hence the multiplicity of the closed string states is given by

这在所有已知案例中都确实成立。因此闭弦态的简并度由下式给出

$$\frac{1}{2}(Z_{ii}+K_i)$$

Note that it is possible for closed string states to be ”projected out,” i.e., removed from the spectrum, if $K_i = -Z_{ii}$.

注意，当 $K_i = -Z_{ii}$ 时，闭弦态可以被「投影出去」，也就是从谱中移除。

The second line in (46) gives rise to open string states, projected by the Moebius amplitude. In this case the coefficients satisfy, in all known theories

(46) 的第二行给出由莫比乌斯振幅投影得到的开弦态。在所有已知理论中，该情况下系数都满足

$$M^i_a = A^i_{aa} \bmod 2 \text{ and } |M^i_a| \leq A^i_{aa}.$$

The natural interpretation is that diagonal states (open strings between the same boundaries) are projected to obtain the following multiplicities:

其自然的解释是，对角态 (相同边界之间的开弦) 经过投影后得到如下简并度:

$$\frac{1}{2}\left(N_a N_a A^i_{aa} + N_a M^a_i\right),$$

whereas the off-diagonal terms are not affected by the Moebius amplitude. In the simplest case, $A^i_{aa}=1$ and $M^a_i = \pm 1$, one gets dimensions of symmetric and antisymmetric tensors. Further work is needed to demonstrate that these particles do indeed couple in that manner.

而非对角项不受莫比乌斯振幅影响。在最简单的情况，即 $A^i_{aa}=1$ 和 $M^a_i=\pm 1$ 时，我们得到对称张量和反对称张量的维数。要证明这些粒子确实按该方式耦合还需要进一步研究。

Gauge Groups

规范群

The gauge bosons come from the identity character. Its massless state is a spacetime vector. In the closed string, a left-right combination of two vectors gives the graviton, the dilaton, and the $B_{\mu\nu}$ Kalb-Ramond field. In the open string sector, the vector comes with multiplicity:

规范玻色子来自恒等特征标，其无质量态是一个时空矢量。在闭弦中，两个矢量的左右组合给出引力子、dilaton 和 $B_{\mu\nu}$ 卡尔布-朗道场。在开弦扇区，矢量具有如下多重度:

$$\frac{1}{2}\left(A^0_{aa}N_aN_a + M^0_aN_a\right).$$

It turns out that the matrix A^0_{ab} in the space of boundaries is a bijection: it has the property that for any a there is precisely one label b with $A^0_{ab} = 1$, and $A^0_{ab} = 0$ for all other b . The label b for which $A^0_{ab} = 1$ is called the complex conjugate boundary, and is denoted a^c . If $a = a^c$, the boundary is called self-conjugate or real. In that case, $M^0_a = \pm 1$. The number of gauge bosons is either $\frac{1}{2}N_a\left(N_a + 1\right)$, suggesting that the gauge group is $USp\left(N_a\right)$, or $\frac{1}{2}N_a\left(N_a - 1\right)$, suggesting that it is $SO\left(N_a\right)$. Further studies of the amplitudes are necessary to verify that this interpretation is correct, because from these arguments we only obtained multiplicities. Note that in the symplectic case, we must have N_a even.

可以证明，边界空间中的矩阵 A^0_{ab} 是双射: 它满足对任意 a ，存在且仅存在一个标记 b 使得 $A^0_{ab} = 1$ 成立，且对所有其他 b 有 $A^0_{ab} = 0$ 。满足 $A^0_{ab} = 1$ 的标记 b 称为复共轭边界，记为 a^c 。若 $a = a^c$ ，该边界称为自共轭边界或实边界，此时 $M^0_a = \pm 1$ 。规范玻色子的数目要么是 $\frac{1}{2}N_a\left(N_a + 1\right)$ ，对应规范群为 $USp\left(N_a\right)$ ；要么是 $\frac{1}{2}N_a\left(N_a - 1\right)$ ，对应规范群为 $SO\left(N_a\right)$ 。需要进一步研究振幅来验证该解释的正确性，因为上述推导仅得到了多重度。注意辛情形下必须满足 N_a 为偶数。

For complex boundaries, those with $a \neq a^c$, we get a vector boson multiplicity $\frac{1}{2}N_aN_a\left(A^0_{aa^c} + A^0_{a^ca}\right) = \left(N_a\right)^2$, since $M^0_a = 0$. This suggests a group $U\left(N_a\right)$, which is indeed correct. Hence now we can get, in principle, the same brane configurations as in the oriented case, but with the additional option of symmetric and anti-symmetric tensors. If b is not equal to a or a^c , we get a bi-fundamental.

对于满足 $a \neq a^c$ 的复边界，由于 $M^0_a = 0$ ，我们得到矢量玻色子多重度 $\frac{1}{2}N_aN_a\left(A^0_{aa^c} + A^0_{a^ca}\right) = \left(N_a\right)^2$ 。这对应群 $U\left(N_a\right)$ ，该结论是正确的。因此原则上我们现在可以得到和定向开弦情形相同的膜构型，同时额外多出对称张量和反对称张量的选项。若 b 不等于 a 也不等于 a^c ，我们得到双基本表示。

Completeness, Integrality, and Sewing Constraints

完备性、整性与缝合约束

Now we have to determine the coefficients B_{ia} and Γ_i . Indeed, the first task is to determine the set of labels a , which determine the set of boundaries, and hence, in geometric language, the number of D-branes at our disposal. The answer is provided by a conjecture called the completeness condition for boundaries, formulated in [85]. It states essentially that B_{ia} is an invertible matrix, so the number of labels a is equal to the number of labels i . The latter number can be inferred from (43). Note that this definition pairs states i with their charge conjugates i^c . The closed string states that propagate in the transverse channel are those with $Z_{ii^c} \neq 0$, and they have multiplicity Z_{ii^c} . Hence the number of labels i , also known as Ishibashi labels, is equal to $\sum_i Z_{ii^c} = \mathrm{Tr}\, ZC$. By the completeness conjecture, the number of labels a is the same. It follows that the boundary coefficients B_{ia} depend on the choice of partition function Z_{ij} of the closed string theory.

现在我们需要确定系数 B_{ia} 和 Γ_i 。实际上首要任务是确定标签集合 a ，该集合决定了边界集合，因此从几何角度来说，也决定了我们可用的 D 膜数量。有一个名为边界完备性条件的猜想给出了答案，该猜想提出于文献 [85]。它的核心表述是 B_{ia} 是一个可逆矩阵，因此标签 a 的数量等于标签 i 的数量。后者可由式 (43) 得到。注意该定义将态 i 和其电荷共轭态 i^c 配对。横通道中传播的闭弦态是满足 $Z_{ii^c} \neq 0$ 的态，它们的重数为 Z_{ii^c} 。因此标签 i (也称为石桥标签) 的数量等于 $\sum_i Z_{ii^c} = \mathrm{Tr}\, ZC$ 。根据完备性猜想，标签 a 的数量与之相等。由此可知边界系数 B_{ia} 依赖于对闭弦理论配分函数 Z_{ij} 的选择。

The boundary and crosscap coefficients are subject to the integrality conditions described above, but more importantly by sewing constraints. These constraints follow from the requirement that different ways to build a Riemann surface from three-point functions by ”sewing” must yield the same answer. See [86] for a discussion of these constraints for closed strings, [87] and [88] for open strings and [89] for unoriented strings.

边界和十字帽系数满足上文所述的整性条件，但更重要的是满足缝合约束。这些约束来自如下要求：通过“缝合”从三点函数构造黎曼曲面的不同方式必须得到相同的结果。关于闭弦的这类约束的讨论见文献 [86]，开弦见 [87] 和 [88]，未定向弦见 [89]。

A Solution for C-diagonal Theories

C 对角理论的一个解

The first solution to these constraints in the oriented case was found by Cardy [90]. He considered the C-diagonal modular invariant, and gave the following formula for the boundary coefficients:

定向情况下这些约束的第一个解由卡迪 (Cardy) 找到 [90]。他考虑了 C 对角模不变量，给出了边界系数的如下公式:

$$B_{ia} = \frac{S_{ia}}{\sqrt{S_{0i}}}, \tag{57}$$

where S is the matrix defined in (38). Note that this explicitly satisfies the completeness condition, because S is a square matrix. Substituting this in (54), we get:

其中 S 是式 (38) 中定义的矩阵。请注意这显然满足完备性条件，因为 S 是一个方阵。将其代入式 (54)，我们得到:

$$A^i{}_{ab} = \sum_m \frac{S^i_m S_{ma} S_{mb}}{S_{m0}} = N^i_{ab}, \tag{58}$$

where in the last step we used the Verlinde formula for fusion multiplicities [91]. This shows that also the integrality conditions are satisfied, because fusion coefficients are integer. In fact, the purpose of Cardy's work was to understand the Verlinde formula, but while doing so he proposed and initiated the field of boundary CFT. This was picked up almost immediately [92] by the Tor Vergata (Rome II) group, who did a lot of pioneering work in this area.

其中最后一步我们使用了融合重数的范林德 (Verlinde) 公式 [91]。这说明整性条件同样也得到满足，因为融合系数是整数。事实上，卡迪研究工作的初衷是理解范林德公式，但在此过程中他提出并开创了边界共形场论领域。这一方向几乎立刻就得到了跟进 [92]，托尔韦尔加塔 (罗马第二大学) 组完成了该领域大量开创性工作。

They also considered unoriented surfaces, a subject not treated by Cardy. After some earlier work, in [93] they proposed the following formula for the crosscap coefficients for the case $Z = C$:

他们还研究了非定向曲面，这是卡迪没有处理过的主题。在早期研究之后，他们在文献 [93] 中给出了 $Z = C$ 情况下交叉帽系数的如下公式:

$$\Gamma_i = \frac{P_{0i}}{\sqrt{S_{0i}}}, \tag{59}$$

which uses the P -matrix introduced above.

该公式用到了上文引入的 P 矩阵。

Simple Current Results

简单流结果

Now attempts started to obtain similar results for general simple current modular invariants. This culminated about a decade later in a completely general formula [94] for all simple current MIPFs. This paper contains references to all the earlier partial results obtained by various groups. The formula for boundary coefficients is

目前人们已经开始尝试推导一般简单电流模不变量的类似结果。大约十年后，这项工作最终得到了适用于所有简单流 MIPF 的完全通式 [94]，本文列出了不同研究组此前得到的所有部分结果的相关参考文献。边界系数公式为

$$B_{(i,J)[j,\psi]} = \sqrt{\frac{|\mathcal{G}|}{|\mathcal{S}_i|\,|\mathcal{C}_j|}} \frac{\alpha(J)\, S^J_{ij}}{\sqrt{S_{0i}}} \psi(J)\,, \tag{60}$$

and the crosscap formula is

而交叉帽公式为

$$\Gamma_{(i,J)} = \frac{1}{|\mathcal{G}|} \sum_{K \in \mathcal{G}} \eta(K) P_{Ki} \delta^{J0}. \tag{61}$$

We only present these results here to give a flavor of what is involved. The simple currents form a subgroup $\mathcal{G}$ under fusion. Multiplicities $Z_{ij} > 1$ occur when a field i is a fixed point of the simple current action: $Ji = i$. The simple currents that fix i for a subgroup of $\mathcal{G}$ is called the stabilizer of i and denoted $\mathcal{S}_i$. Consequently, the Ishibashi labels (i, J) are equipped with a degeneracy label $J \in \mathcal{S}_i$, and i is a label such that $Z_{ii^c} = 1$. The boundary states are labeled by orbit representatives i (i.e., one representative of each orbit $(i, Ji, J^2 i, \ldots)$), and their degeneracy is governed by the size of another discrete group, $\mathcal{C}_j \subset \mathcal{S}_j$. The discrete group characters $\psi(J)$ are used as degeneracy labels for the boundaries. The matrices S^J are modular transformation matrices of a "fixed-point CFT," an algebraic structure associated with the fixed points, intuitively introduced in [95] and more rigorously defined in [96]. For further details, such as the precise definition of $\mathcal{C}_j$, the phase $\alpha(J)$, and the signs $\eta(J)$, we refer the reader to [94].

我们在此仅展示这些结果，以帮助读者了解相关内容。简单流在融合乘积下构成子群 $\mathcal{G}$ 。当场 i 是简单流作用的不动点时会出现多重度 $Z_{ij} > 1 : Ji = i$ 。固定 i 的简单流是 $\mathcal{G}$ 的一个子群，称为 i 的稳定子，记为 $\mathcal{S}_i$ 。因此，石桥标签 (i, J) 带有简并标签 $J \in \mathcal{S}_i$ ，且 i 是满足 $Z_{ii^c} = 1$ 的标签。边界态由轨道代表元 i 标记 (即每个轨道 $(i, Ji, J^2 i, \ldots)$ 取一个代表元)，其简并由另一个离散群 $\mathcal{C}_j \subset \mathcal{S}_j$ 的阶决定。离散群特征标 $\psi(J)$ 被用作边界的简并标签。矩阵 S^J 是“不动点共形场论”的模变换矩阵，不动点共形场论是与不动点相关的代数结构，[95] 给出了直观介绍，[96] 给出了更严格的定义。关于 $\mathcal{C}_j$ 的精确定义、相位 $\alpha(J)$ 以及符号 $\eta(J)$ 等更多细节，请读者参考文献 [94]。

The boundary coefficients are uniquely defined as soon as the torus partition function Z_{ij} is known, but for the crosscap there is a variety of possibilities. First of all there is a "Klein bottle current" K , and secondly the signs $\eta(J)$ must satisfy a condition that may have several solutions. Taking into account these choices for different crosscaps increases the total number of distinct, unoriented Gepner CFT models to 49304. However only 33012 of them have nonzero tension. The remaining ones are not usable for building supersymmetric orientifold models.

只要环面配分函数 Z_{ij} 已知，边界系数就能被唯一确定，但交叉帽存在多种可能性。首先存在一个“克莱因瓶流” K ，其次符号 $\eta(J)$ 需要满足的条件可能存在多个解。将这些不同交叉帽的选择考虑在内后，不同的无定向格普纳共形场论模型总数增加到了 49304 个。但其中只有 33012 个模型具有非零张力，剩下的模型无法用于构建超对称定向闭曲面模型。

RCFT Model Building

RCFT 模型构建

We are now ready to apply this machinery to model building. In the situation of interest, the relevant CFT is the usual worldsheet theory of the superstring (or non-supersymmetric fermionic string) in four flat dimensions, combined with a non-trivial superconformal CFT to describe the six "compactified" dimensions. The characters are products of superstring characters and internal characters. The former determine the space-time properties of the string excitations, in particular spin and chirality, whereas the latter contribute to the counting of states. The gauge representations of the physical states can be read off from the Chan-Paton labels of the string state under consideration.

现在我们已经可以将这套方法应用于模型构建了。在我们关注的情形中，相关共形场论 (CFT) 是四维平坦空间中超弦 (或非超对称费米弦) 的常规世界面理论，结合一个非平凡超共形 CFT 来描述六个“紧致化”维度。特征标是超弦特征标与内部特征标的乘积。前者决定弦激发的时空性质，尤其是自旋和手性，后者则用于态计数。物理态的规范表示可以从所考虑弦态的陈-帕顿标签直接读出。

Now we start with a choice of an RCFT (in practice always a Gepner model), a choice of a MIPF, a choice of a crosscap, and a choice of three or four boundary labels (a, b, c) and d , depending on the configuration we want to realize, as discussed in section ”The Simplest Examples.” Usually a is taken to be the QCD label and b is the weak label. We can already make sure that $N_a = 3$ and $N_b = 2$ do not oversaturate the dilaton tadpole. Now we compute the annulus coefficients $A^i_{ab}, A^i_{ac}, A^i_{bc}$, etc. for any two chosen labels and check if the chiral intersections (as defined in Table 1) match the required spectrum.

现在我们开始依次选择 RCFT(实际中总是 Gepner 模型)、MIPF、crosscap，再根据我们想要实现的构型选择三个或四个边界标签 (a, b, c) 和 d ，正如“最简单的例子”一节讨论的那样。通常 a 取 QCD 标签，b 取弱相互作用标签。我们已经可以确认 $N_a = 3$ 和 $N_b = 2$ 不会让 dilaton 蝌蚪过饱和。接下来我们计算任意两个所选标签的环面系数 $A^i_{ab}, A^i_{ac}, A^i_{bc}$ 等，检查手性交叉 (定义见表 1) 是否符合要求的能谱。

In case of success, we now check if the putative Y boson remains massless after taking into account axion mixing. This requires checking all axions in the full closed string spectrum. At the same time, we may check if any other $U(1)$ vector bosons in the spectrum acquire a mass. If they do not, this is a phenomenological issue which we will have to deal with later because it is not solved at the level of the RCFT.

如果检查通过，我们接下来要确认假定的 Y 玻色子在考虑轴子混合后仍然保持无质量。这需要检查完整闭弦能谱中的所有轴子。同时我们可以检查能谱中其他所有 $U(1)$ 矢量玻色子是否获得质量。如果它们没有获得质量，这就是一个需要我们后续处理的唯象学问题，因为该问题无法在 RCFT 层面得到解决。

The next step is to cancel all tadpoles by finding a suitable hidden sector, as explained in section ”Anomalies, Tadpoles, and Axions.” This can be very time-consuming, because it requires considering all subsets of all branes not used in the Standard Model configuration. In practice, this can usually not be done exhaustively.

下一步是按照“反常、蝌蚪与轴子”一节中的说明，寻找合适的隐层抵消所有蝌蚪。这个过程非常耗时，因为需要考虑标准模型构型之外所有膜的所有子集。实际中通常无法完成穷举搜索。

There is still one more check to be made, the absence of global anomalies, as discussed in [97].

还需要做最后一项检查: 不存在整体反常，相关讨论见文献 [97]。

The first work along these lines appeared in [98]. In this paper six-dimensional theories were studied, and examples with chiral spectra were found. In [99], building on [100], the first chiral spectra were found in four dimensions. Then in [101, 102] a general search was undertaken for the Madrid configuration shown in Fig. 4 and some of its variations.

该方向的第一项工作发表于文献 [98]。该文研究了六维理论，找到了手性能谱的例子。文献 [99] 在文献 [100] 的基础上，首次在四维中找到了手性能谱。随后文献 [101, 102] 对图 4 所示的马德里构型及其部分变体开展了通用搜索。

In [18] a different approach was taken. Rather than searching for specific preselected brane configurations, these authors searched for any combinations of brane labels that yields the Standard Model, in a rather generous definition of the latter. This includes enlarged gauge groups, non-chiral pairs of quarks and leptons as chiral brane matter (i.e., chiral matter that becomes non-chiral if only the SM group is considered) and gauged flavor symmetries. The result essentially demonstrated that anything one could theoretically propose as a Standard Model configuration will likely be realizable if one has a large enough scope of brane models to start with.

文献 [18] 采用了不同的方法。这些作者没有搜索预先选定的特定膜构型，而是按照对标准模型相当宽泛的定义，搜索所有能得到标准模型的膜标签组合。该定义包含扩大的规范群、作为手性膜物质的夸克轻子非手性对 (即仅考虑 SM 群时会变为非手性的手性物质) 以及定味 flavour 对称性。结果基本表明，如果起始的膜模型范围足够大，理论上能提出的任何标准模型构型基本都可以实现。

One may think that this scope can be extended significantly by moving to other RCFTs, but in practice this is not easy. For most RCFTs we simply do not have all the relevant data available. It is often easy to get the spectrum modulo integers, but a lot harder to get the exact spectrum, as required. Furthermore any RCFT building blocks other than $\mathcal{N} = 2$ minimal models (one example are the Kazama-Suzuki models [103]) have much smaller simple current groups, and hence far fewer MIPFs. There might exist vast numbers of exceptional (not simple current related) MIPFs in some cases, but a general formalism to compute their boundary and crosscap coefficients is not available. Free fermion orientifolds have been considered, but are a far less fertile area [104]. So it seems that Gepner models are in a sense the optimal possibility.

有人或许认为转向其他 RCFT 可以大幅拓展这个范围，但实际中这并不容易。对于大多数 RCFT，我们目前根本没有所有相关数据。通常得到模整数后的能谱很容易，但得到我们需要的精确能谱要困难得多。此外，除了 $\mathcal{N} = 2$ 极小模型外，其他任何 RCFT 结构块 (一个例子是 Kazama-Suzuki 模型 [103]) 的单流群都小得多，因此 MIPF 的数量也少得多。某些情况下可能存在大量例外 (与单流无关)MIPF，但目前没有计算其边界和 crosscap 系数的通用形式。自由费米子 orientifold 已经被研究过，但可得到的结果远更少 [104]。因此从某种意义上说，Gepner 模型似乎是最优选择。

F-Theory Model Building

F 理论模型构建

F-theory was introduced in [105] as a non-perturbative formulation of Type IIB compactifications with 7-branes. Its importance for model building is owed to its generality, including the fact that it allows for the construction of gauge sectors which enjoy an embedding into the exceptional Lie group E_8 . This property makes it a particularly natural framework to study Grand Unified Theories (GUTs) in string theory and distinguishes it from its perturbative cousins discussed in the previous sections, which are based on gauge groups $U(N), SO(N)$, and $USp(N)$. Hence, F-theory combines the attractive features of model building with D-branes - the localization of gauge degrees of freedom on branes, which in principle invite a local approach to model building within a certain realm of questions - with the appearance of exceptional gauge symmetry as in heterotic $E_8 \times E_8$ string theory, which bears its fruit in the context of GUT model building. The goal of this program is to solve some of the outstanding model building challenges faced by four-dimensional SUSY GUTs in the higher-dimensional brane-world framework provided by F-theory.

F 理论于文献 [105] 中被提出，是带 7 膜的 IIB 型紧致化的非微扰表述。它对模型构建的重要性源于其通用性，其中一点是它允许构造可嵌入例外李群 E_8 的规范 sector。这一性质使它成为弦论中研究大统一理论 (GUT) 格外自然的框架，并区别于前几节讨论的基于规范群 $U(N)$, $SO(N)$ 和 $USp(N)$ 的微扰同类理论。因此，F 理论结合了 D 膜模型构建的吸引人特征——规范自由度定域在膜上，原则上允许对特定范围内的问题采用局域方法构建模型——也拥有杂化 $E_8 \times E_8$ 弦论中那样的例外规范对称性，这一大特性在 GUT 模型构建中十分有用。该研究方向的目标是，在 F 理论提供的高维膜世界框架下，解决四维超对称 GUT 面临的部分突出的模型构建难题。

In addition, F-theory offers a formulation of D-branes in terms of the geometry of so-called elliptic (or more generally genus-one) fibrations; many involved questions of brane dynamics are hence translated into entirely geometric questions and oftentimes have a clear answer in algebraic or arithmetic geometry.

此外，F 理论借助所谓椭圆 (更一般地是亏格一) 纤维化的几何给出了 D 膜的表述；许多复杂的膜动力学问题因此被转化为纯几何问题，且往往能在代数几何或算术几何中得到明确解答。

General introductions to F-theory are provided for instance in [106-111], to which we refer for details and the original references. In the sequel, after briefly presenting some of the technical foundations of F-theory, we focus on its role for particle physics model building.

例如文献 [106-111] 提供了 F 理论的通用介绍，更多细节和原始参考文献可参阅这些文献。下文在简要介绍 F 理论的部分技术基础后，将聚焦它在粒子物理模型构建中的作用。

From [p, q] 7-Branes to Exceptional Gauge Algebras

从 [p, q] 7-膜到例外规范代数

The starting point for F-theory is Type IIB string theory. Type IIB string theory contains in its massless spectrum higher-form Ramond-Ramond (RR) gauge potentials of even degree, C_{2p} , with $p = 0, \ldots, 4$. The 2-form C_2 couples electrically to a D1-brane, which is a stringlike soliton of tension $T_{\mathrm{D1}} = \frac{2\pi}{\ell_s^2}\frac{1}{g_s}$ in perturbative string theory. The Neveu-Schwarz 2-form potential B_2 , on the other hand, couples electrically to the fundamental, or F1− , string of tension $T_{\mathrm{F1}} = \frac{2\pi}{\ell_s^2}$. These two types of strings can form BPS-bound states: A bound state of p F1-strings and q D1- branes is called a (p, q) string, and it exists as a BPS-bound state for p and q co-prime integers. Type IIB string theory enjoys a weak-strong coupling duality, which maps an F1- or $(1, 0)$ -string into a D1- or $(0, 1)$ -string, and vice versa. The theory is believed to be invariant, at the non-perturbative level, under an $SL(2, \mathbb{Z})$ duality transformation, which in particular acts on the axio-dilaton $\tau = C_0 + \frac{i}{g_s}$ and the two-form potentials as

F 理论的出发点是 IIB 型弦论。IIB 型弦论的无质量谱包含偶数次的高阶拉姆 ond-拉姆 ond(RR) 规范势 C_{2p}，满足 $p = 0, \ldots, 4$。2 形式 C_2 与 D1-膜电耦合，D1-膜是微扰弦论中张力为 $T_{\mathrm{D1}} = \frac{2\pi}{\ell_s^2} \frac{1}{g_s}$ 的弦状孤子。另一方面，纳伏-施瓦茨 2 形式势 B_2 与张力为 $T_{\mathrm{F1}} = \frac{2\pi}{\ell_s^2}$ 的基本弦 (即 F1−) 电耦合。这两类弦可以形成 BPS 束缚态: p 根 F1 弦与 q 个 D1-膜组成的束缚态称为 (p, q) 弦，当 p 和 q 为互素整数时，该 BPS 束缚态存在。IIB 型弦论具有强弱耦合对偶性，可将 F1 弦 (即 $(1, 0)$ 弦) 映射为 D1 弦 (即 $(0, 1)$ 弦)，反之亦然。一般认为该理论在非微扰层面具有 $SL(2, \mathbb{Z})$ 对偶变换下的不变性，该变换对轴子- dilaton $\tau = C_0 + \frac{i}{g_s}$ 和两类 2 形式势的作用具体为

$$\tau \to \frac{a\tau + b}{c\tau + d}, \begin{pmatrix} C_2 \\ B_2 \end{pmatrix} \to M \begin{pmatrix} C_2 \\ B_2 \end{pmatrix}, M = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2, \mathbb{Z}). \tag{62}$$

Of central interest for the formulation of F-theory are the Type IIB 7-branes. A D7-brane, often called [1, 0]-brane in the sequel, is, by definition, a 7-brane on which an F1-string can end; $SL(2, \mathbb{Z})$ duality implies that there must then exist also another type of 7-brane, a so-called $[0, 1]$ -brane, on which a D1-string can end. More generally one defines a $[p, q]$ 7-brane as a 7-brane on which a (p, q) -string can end.

构建 F 理论的核心研究对象是 IIB 型 7-膜。D7-膜在后文常记作 $[1, 0]$-膜，根据定义它是 F1 弦可以终止于其上的 7-膜；$SL(2, \mathbb{Z})$ 对偶意味着必然存在另一类 7-膜，即所谓的 $[0, 1]$ -膜，D1 弦可以终止于其上。更一般地，我们将 $[p, q]$ 7-膜定义为 (p, q) 弦可以终止于其上的 7-膜。

The fact that a 7-brane is an object of real codimension two in ten dimensions implies a rather severe backreaction on the supergravity background due to its tension and RR charge. If one considers a single D7-brane and introduces the complex coordinate $z = x^8 + ix^9$ in the complex plane spanned by the two directions normal to the brane, the axio-dilaton τ acquires a varying profile which close to the brane at $z = 0$ can be approximated as

7-膜是十维中余维数为 2 的客体,这一特点意味着它的张力和 RR 荷会对超引力背景产生相当显著的反作用。如果我们考虑单个 D7-膜,并在垂直于膜的两个方向张成的复平面中引入复坐标 $z = x^8 + ix^9$，轴子-dilaton τ 会形成变化的分布，在靠近 $z = 0$ 处的膜附近，该分布可以近似为

$$\tau(z) = \tau_0 + \frac{1}{2\pi i} \log\left(\frac{z}{z_0}\right). \tag{63}$$

Due to the logarithmic branch cut, $\tau(z)$ undergoes a shift $\tau \to \tau + 1$ as one encircles the D7-brane at $z = 0$. This multi-valuedness is consistent because it is accompanied by an $SL(2, \mathbb{Z})$ duality transformation on all the Type IIB fields, which merely accounts for a change of duality frame. The $SL(2, \mathbb{Z})$ monodromy induced in this way by a single $[p, q]$ 7-brane can be represented by the action of a monodromy matrix of the form

由于对数分支切割的存在，当我们环绕 $z = 0$ 处的 D7-膜一周时，$\tau(z)$ 会发生偏移 $\tau \to \tau + 1$。这种多值性是自洽的，因为它伴随所有 IIB 型场的 $SL(2, \mathbb{Z})$ 对偶变换，这仅仅对应对偶框架的改变。单个 $[p, q]$ 7-膜以此方式诱导的 $SL(2, \mathbb{Z})$ 单值群可以用如下形式的单值矩阵作用表示

$$M_{[p,q]} = \begin{pmatrix} 1 - pq & p^2 \\ -q^2 & 1 + pq \end{pmatrix}. \tag{64}$$

Two $[p, q]$ 7-branes are called mutually nonlocal if their monodromy matrices cannot be brought into the same form by an $SL(2, \mathbb{Z})$ transformation.

如果两个 $[p, q]$ 7-膜的单值矩阵无法通过同一个 $SL(2, \mathbb{Z})$ 变换化为相同形式，则称它们是互不对易的。

Not all types of mutually nonlocal $[p, q]$ 7-branes can be consistently placed on top of each other to form a supersymmetric brane stack without leading to a drastic degeneration of the theory such as an effective decompactification. Let us first consider 7-branes in flat space $\mathbb{R}^{1,9}$. In order to describe the allowed configurations of coincident $[p, q]$ 7-branes in 10 dimensions, it suffices to consider three types of mutually nonlocal $[p, q]$ 7-branes. A possible such generating system consists of the following three brane types:

并非所有类型的互非局域 $[p, q]$ 7 膜都能稳定地堆叠在一起形成超对称膜堆，同时不引发理论的剧烈退化, 例如有效退紧致化。我们先来考虑平直空间 $\mathbb{R}^{1,9}$ 中的 7 膜。要描述 10 维中允许的重合 $[p, q]$ 7 膜构型，只需考虑三类互非局域的 $[p, q]$ 7 膜。一个可用的生成系由以下三类膜组成:

$$A : [p, q] = [1, 0]\,,\ B : [p, q] = [3, -1]\,,\ C : [p, q] = [1, -1]\,. \tag{65}$$

In the chosen duality frame, an A-type brane corresponds to a perturbative D7- brane. The O7-plane from Type IIB orientifolds is realized as a BC -brane system. Consistently the monodromy matrix $M_{BC} = M_B M_C$ acts as multiplication with (-1) on a $(1, 0)$ string, which is interpreted as worldsheet parity. In this way one recovers the perturbative gauge groups $SU(N)$ (omitting the diagonal $U(1)$ factor) by a configuration of NA -type branes, while $SO(N)$ groups correspond to a system of NA -type branes on top of a BC brane system, i.e., on an O7-plane (For $N < 4$, non-perturbative effects lead to a dynamical separation of the branes.). The most important observation, however, is that in addition to these perturbative gauge groups, also the exceptional gauge groups E_N for $N = 6, 7, 8$ can be obtained in flat space from brane configurations of the form $A^{N-1}BCC$.

在我们选取的对偶框架中，A 型膜对应微扰 D7 膜。IIB 型定向模中的 O7 平面可实现为一个 BC 膜系统。单值矩阵 $M_{BC} = M_B M_C$ 对 $(1, 0)$ 弦的作用确实是乘以 (-1)，这对应世界面宇称。借此，NA 型膜的构型可以重现微扰规范群 $SU(N)$ (省略对角 $U(1)$ 因子)，而 $SO(N)$ 群对应位于 BC 膜系统即 O7 平面之上的 NA 型膜系统 (对于 $N < 4$ ，非微扰效应会引发膜的动力学分离)。但最重要的结论是: 除这些微扰规范群外，平直空间中 $A^{N-1}BCC$ 形式的膜构型也可以得到 $N = 6, 7, 8$ 对应的例外规范群 E_N 。

The emergence of exceptional gauge algebras is rooted in the possible $[p, q]$ - strings stretched between the mutually nonlocal branes in the configuration. These include, in the present setup, bound states of $[p, q]$ -strings with three endpoints - so-called multipronged strings - which end on three mutually nonlocal branes, rather than just two branes of the same type as in perturbative setups. A careful analysis of the possible multipronged strings [112] for the configuration $A^{N-1}BCC$ indeed identifies all the roots of the exceptional series $E_N, N = 6, 7, 8$.

例外规范代数的出现源于构型中互非局域膜之间拉伸的 $[p,q]$ 弦。在当前框架下，这些弦包括带有三个端点的 $[p,q]$ 弦束缚态——即所谓多叉弦——它们终止于三个互非局域膜，而非微扰框架中仅终止于两个同类型膜。对构型 $A^{N-1}BCC$ 可能的多叉弦的细致分析 [112] 确实识别出例外序列 $E_N, N=6,7,8$ 的所有根。

The easiest way to classify the allowed configurations of coincident 7-branes is by passing to the geometrized description of F-theory as pioneered in [105, 113 , 114]. The behavior of the axio-dilaton τ under an $SL(2,\mathbb{Z})$ transformation,(62), is reminiscent of the transformation of the modular parameter of an elliptic curve under its modular group. This motivates interpreting τ as the complex structure parameter of an elliptic curve $\mathbb{E}_\tau$, which varies holomorphically in the directions normal to the 7-brane according to (63). This structure defines an elliptic, or more generally a genus-one [115] (The difference is that an elliptic fibration necessarily has a section. In the sequel we will, for simplicity, use the term elliptic fibration for both constructions.), fibration over the directions normal to the branes.

对允许的重合 7 膜构型分类的最简单方法，是采用 [105, 113 [114] 中开创的 F 理论几何化描述。轴子- dilation 场 τ 在 $SL(2,\mathbb{Z})$ 变换 (62) 下的行为，让人联想到椭圆曲线模参数在模群下的变换性质。这启发我们将 τ 解释为椭圆曲线 $\mathbb{E}_\tau$ 的复结构参数，该参数根据 (63) 沿垂直于 7 膜的方向全纯变化。这个结构定义了一个在垂直于膜方向上的椭圆纤维化，更一般地是亏格一纤维化 [115](二者的区别在于椭圆纤维化一定存在一个截面。为简化起见，下文中我们对两种构造都使用椭圆纤维化这一术语。)。

The simplest manifestation of this idea is to consider compactifications of F-theory to eight dimensions with general $[p,q]$ 7-branes filling the uncompactified dimensions $\mathbb{R}^{1,7}$. Due to the backreation of the 7-branes, the two compact normal directions must be curved to form a projective sphere $\mathbb{P}^1$. The variation of the axio-dilaton over $\mathbb{P}^1$ can be identified with the variation of the complex structure parameter of an elliptic curve fibered over $\mathbb{P}^1$. Supersymmetry requires that the elliptic fibration defined in this fashion is a Calabi-Yau manifold, and in this case an elliptically fibered K3 surface. The physical compactification space from ten to eight dimensions, $\mathbb{P}^1$, represents the base of this elliptic fibration. Note that this physical compactification space is not Calabi-Yau, but rather carries positive curvature as a result of the backreaction. The $[p,q]$ 7-branes sit at special points on the base $\mathbb{P}^1$ over which the complex structure τ of the elliptic fiber $\mathbb{E}_\tau$ degenerates to reflect the singular behavior of the axio-dilaton profile (63) for $z \to 0$. The monodromies around the location of a $[p,q]$ 7-brane have a direct geometric interpretation as monodromies that transform a local basis of one-cycles on the elliptic fiber into one another as one transports it around the 7-brane location.

这一想法最简单的体现是，考虑 F-theory 紧致化到八维，其中一般 $[p,q]$ 7-膜充满未紧致维度 $\mathbb{R}^{1,7}$ 。由于 7-膜的反作用，两个紧致法向方向必须弯曲形成射影球面 $\mathbb{P}^1$ 。轴子- dilaton 场在 $\mathbb{P}^1$ 上的变化，等同于纤维化在 $\mathbb{P}^1$ 上的椭圆曲线的复结构参数的变化。超对称要求，以此方式定义的椭圆纤维化是一个卡拉比-丘流形，在该情形下是椭圆纤维化 K3 曲面。从十维紧致化到八维得到的物理紧致化空间 $\mathbb{P}^1$ ，就是该椭圆纤维化的底空间。注意这个物理紧致化空间并不是卡拉比-丘流形，反作用反而会给它带来正曲率。$[p,q]$ 7-膜位于底空间 $\mathbb{P}^1$ 的特殊点上，在这些点椭圆纤维 $\mathbb{E}_\tau$ 的复结构 τ 发生退化，对应 $z \to 0$ 轴子-dilaton 分布的奇异行为 (63)。当我们环绕 7-膜位置移动局部 1 基圈时，基圈会发生相互变换，因此 $[p,q]$ 7-膜位置周围的单极群可以直接被几何解释为该变换对应的单极群。

The possible types of monodromies that can consistently occur in this manner in an effectively eight-dimensional compactification have been classified geometrically by Kodaira and Néron [116-118] and are in one-to-one correspondence with A-D-E type singularities of the elliptic fiber. By interpreting a monodromy as a product of monodromy matrices of different $[p,q]$ 7-branes, one can translate this geometric classification into a classification of allowed stacks of coincident 7-branes on $\mathbb{P}^1$.

这种在有效八维紧致化中能够自洽出现的单极群类型，已经由 Kodaira 和 Néron 从几何上完成分类 [116-118]，且它们与椭圆纤维的 A-D-E 型奇点存在一一对应关系。将整体单极群解读为不同 $[p,q]$ 7-膜的单极矩阵的乘积，我们就可以把这个几何分类转换为 $\mathbb{P}^1$ 上允许的重合 7-膜堆的分类。

The classification starts from the notion of a Weierstrass model for the elliptic fibration: an elliptic curve $\mathbb{E}_\tau$ with modular parameter τ can be represented as the hypersurface:

该分类从椭圆纤维化的魏尔斯特拉斯模型概念出发: 带模参数 τ 的椭圆曲线 $\mathbb{E}_\tau$ 可以表示为超曲面:

$$P_W \ := \ y^2 - \left(x^3 + fxz^4 + gz^6\right) = 0 \tag{66}$$

Table 7 Kodaira table for singular fibers of an elliptic K3 [116-118]. The non-minimal enhancements in the last row can lie at finite or infinite distance [119]; in the latter case, one can associate a loop algebra to the singularity

表 7 椭圆 K3 的奇异纤维的 Kodaira 表 [116-118]。最后一行的非极小增强可以位于有限或无限距离处 [119]；在后一种情形中，可以给该奇点关联一个圈代数。

type	ord (f)	ord (g)	ord (Δ)	sing.	G
I_0	⍰0	⍰0	0	-	-
I_1	0	0	1	-	-
II	⍰1	1	2	-	-
III	1	⍰2	3	A_1	SU(2)
IV	⍰2	2	4	A_2	SU(3)
I_m	0	0	m	A_{m-1}	SU(m)
I^*_{m-6}	2	3	m	D_{m-2}	$SO(2m-4)$
IV^*	⍰3	4	8	E_6	E_6
III*	3	⍰5	9	E_7	E_7
II*	⍰4	5	10	E_8	E_8
non-min	⍰4	⍰6	⍰12		

in the weighted projected space $\mathbb{P}_{2,3,1}$ with homogenous coordinates $[x:y:z]$. Here f and g are complex parameters which determine the complex structure or modular parameter τ as

位于加权射影空间 $\mathbb{P}_{2,3,1}$ 中，齐次坐标为 $[x:y:z]$ 。此处 f 和 g 是确定复结构或模参数 τ 的复参数，满足

$$j(\tau) = \frac{4(24f)^3}{\Delta}, \ \Delta = 4f^3 + 27g^2, \tag{67}$$

where $j(\tau)$ is the $SL(2,\mathbb{Z})$ invariant Jacobi j -function. When the discriminant Δ vanishes, the elliptic curve $\mathbb{E}_\tau$ degenerates. This description is promoted to a Weierstrass model for the elliptically fibered K3 surface by allowing f and g - and hence τ - to vary suitably over the base $\mathbb{P}^1$ (Since $\mathbb{P}^1$ is compact, f and g cannot be globally defined functions, but must rather represent sections of a certain line bundle, whose degree is fixed uniquely by the requirement that the elliptic fibration be Calabi-Yau. In fact, f and g must be sections of $\bar{K}^4_{\mathbb{P}^1}$ and $\bar{K}^6_{\mathbb{P}^1}$, respectively, where $\bar{K}_{\mathbb{P}^1} = \mathcal{O}_{\mathbb{P}^1}(2)$ is the anti-canonical bundle on $\mathbb{P}^1$.). The possible types of singularities in the elliptic fiber are classified as in Table 7.

其中 $j(\tau)$ 是 $SL(2,\mathbb{Z})$ 不变雅可比 j 函数。当判别式 Δ 为零时，椭圆曲线 $\mathbb{E}_\tau$ 发生退化。通过令 f 和 g (进而令 τ) 在底空间 $\mathbb{P}^1$ 上适当变化，该描述被推广为椭圆纤维化 K3 曲面的魏尔斯特拉斯模型 (由于 $\mathbb{P}^1$ 是紧致的，f 和 g 不能是全局定义的函数，而必须表示为某特定线丛的截面，该线丛的次数由椭圆纤维化是卡拉比-丘流形这一要求唯一确定。事实上，f 和 g 必须分别是 $\bar{K}^4_{\mathbb{P}^1}$ 和 $\bar{K}^6_{\mathbb{P}^1}$ 的截面，其中 $\bar{K}_{\mathbb{P}^1} = \mathcal{O}_{\mathbb{P}^1}(2)$ 是 $\mathbb{P}^1$ 上的反典范丛。)。椭圆纤维的可能奇点类型已在表 7 中分类。

F-Theory on Elliptic Fourfolds

椭圆四维流形上的 F 理论

This geometric classification can be extended to higher-dimensional elliptic fibra-tions. Relevant for compactifications of F-theory to four dimensions are elliptic Calabi-Yau fourfolds Y_4 . These automatically preserve $4d\mathcal{N} = 1$ supersymmetry at the geometric level. From the Type IIB perspective, the base B_3 of the elliptic fibration represents the physical compactification space, while the additional two directions along the torus fiber keep track of the type and location of the 7-branes. In this sense, B_3 takes the role of the compactification space X_6 of section "Type

这种几何分类可以推广到高维椭圆纤维化。对 F 理论四维紧致化最相关的是椭圆卡拉比-丘四维流形 Y_4 。这类结构在几何层面自动保持 $4d\mathcal{N} = 1$ 超对称。从 IIB 型弦论视角看，椭圆纤维化的底空间 B_3 是物理紧致化空间，而环面纤维额外的两个方向记录了 7-brane 的种类和位置。在此意义下，B_3 承担了“IIB 型定向轨形”一节中紧致化空间 X_6 的作用，

IIB Orientifolds," including the effect of the orientifold action (The subscript in B_3 refers to its complex dimension.). As was the case for the base of an elliptic K3 surface, B_3 has positive curvature due to the 7-brane backreaction.

包含了定向轨形作用的效应 (B_3 的下标对应其复维数)。和椭圆 K3 曲面底空间的情况一样，由于 7-brane 的背景反应，B_3 具有正曲率。

Such four-dimensional F-theory compactifications depend on two types of data - the geometry of the elliptic fibration and additional gauge backgrounds.

这类四维 F 理论紧致化依赖两类数据——椭圆纤维化的几何结构，以及额外的规范背景。

Geometric Data: Gauge Group, Matter, and Couplings

几何数据: 规范群、物质与耦合

The nature of F-theory as a theory of intersecting 7-branes is reflected in the singularity structure of the elliptic fiber of Y_4 along strata of codimension one, two, and three on the base B_3 :

F 理论作为相交 7 膜理论的性质，反映在 Y_4 的椭圆纤维沿底空间 B_3 上余维 1、2、3 层的奇点结构中:

1. Non-abelian gauge algebras from codimension-one singularities: The discriminant locus Δ on B_3 is defined as the vanishing locus of the combination $4f^3+27g^2$ of the sections f and g that enter the definition of the Weierstrass model. It represents a holomorphic four-cycle on B_3 which is identified with the cycle wrapped by the 7-branes. In general, the 4-cycle class of Δ can be decomposed as

1. 余维 1 奇点给出非阿贝尔规范代数: B_3 上的判别轨迹 Δ 定义为进入魏尔斯特拉斯模型定义的截面 f 与 g 的组合 $4f^3+27g^2$ 的零点轨迹。它对应 B_3 上的全纯四维闭链，等同于 7 膜缠绕的闭链。一般情况下，Δ 的四维闭链类可分解为

$$[\Delta]=\sum_i a_i\left[\sum\nolimits_i\right]+[\Delta'] \tag{68}$$

where the holomorphic four-cycle $\sum_i$ carries a stack of 7-branes associated with a vanishing order a_i in Table 7, while $[\Delta']$ is the remaining piece of the discriminant with vanishing order 1 . The non-abelian gauge algebra G_i associated with $\sum_i$ can be read off from Table 7, where in addition one has to take into account monodromies [120-122] along $\sum_i$ which can lead to smaller gauge algebras than on K3, including all non-simply laced ones. As in the Type IIB orientifold context, the inverse gauge coupling squared associated with G_i is set by the volume of the wrapped cycle $\sum_i$.

其中全纯四维闭链 $\sum_i$ 携带一堆 7 膜，对应表 7 中零点阶数 a_i ，而 $[\Delta']$ 是判别式中剩余的零点阶数为 1 的部分。可从表 7 读出与 $\sum_i$ 关联的非阿贝尔规范代数 G_i ，此外还必须考虑沿 $\sum_i$ 的单值群 [120-122]，单值群会得到比 K3 更小的规范代数，包含所有非 simply-laced(非 simply 单连通根系) 规范代数。和 IIB 型定向模背景一样，与 G_i 关联的逆规范耦合平方由被缠绕闭链 $\sum_i$ 的体积决定。

2. Localized matter from codimension-two singularities: Over special curves $C_{\mathbf{R}}$, extra charged massless matter multiplets reside which transform in some representation $\mathbf{R}$ of the gauge algebra. These curves $C_{\mathbf{R}}$ correspond to the intersection loci of the 7-branes, including possible self-intersections. The charged massless matter fields arise from open (p,q) -strings stretched between the intersecting 7-branes. One can systematically identify the matter curves by searching for enhancements in the vanishing orders of the Weierstrass model data, which signal an enhancement in the singularity structure of the elliptic fiber [122, 123].

2. 余维 2 奇点给出局域化物质: 在特殊曲线 $C_{\mathbf{R}}$ 上存在额外带荷无质量物质多重态，它们变换在规范代数的某个表示 $\mathbf{R}$ 下。这些曲线 $C_{\mathbf{R}}$ 对应 7 膜的相交轨迹，包含可能的自相交。带荷无质量物质场来自拉伸在相交 7 膜之间的开 (p,q) 弦。我们可以通过寻找魏尔斯特拉斯模型数据中零点阶数的提升来系统识别物质曲线，这种提升预示着椭圆纤维奇点结构的增强 [122, 123]。

3. Yukawa couplings from codimension-three singularities: At the intersection of two or more matter curves, Yukawa interactions between the matter multiplets are localized [124-128]. The strength of the couplings depends on the overlap of the wavefunctions of the participating massless modes. The physics rationale behind these couplings is completely analogous to the Type II orientifold setting (see section ”Yukawa Couplings”). Geometrically, to each such Yukawa point, one can associate a higher singularity type in the fiber, and the resulting Yukawa couplings follow from the group theoretically allowed triple couplings within the associated higher symmetry group. Note that the higher singularity types characterizing the singularity enhancements over curves and points do not correspond to gauge algebras in the four-dimensional effective action.

3. 余维 3 奇点给出汤川耦合: 在两条或多条物质曲线的交点处，物质多重态之间的汤川相互作用发生局域化 [124-128]。耦合强度由参与相互作用的无质量模波函数的交叠决定。这些耦合的物理原理与 II 型定向模背景完全类似 (见“汤川耦合”小节)。几何上，每个这类汤川点都对应纤维中更高阶的奇点类型，得到的汤川耦合满足关联高阶对称群中群论允许的三耦合。注意，刻画曲线和点上奇点增强的高阶奇点类型并不对应四维有效作用量中的规范代数。

The discussion so far has focused on the non-abelian part of the gauge algebra and its charged matter and their Yukawa type couplings. This data can, to a certain extent, be described already locally by analyzing the gauge theory along a stack of 7-branes together with additional matter on curves where other branes intersect the brane stack. The local approach amounts to zooming into the neighborhood of one of the components $\sum_i$ of the discriminant. This is analogous to the local approach described in the perturbative context in section ”Type IIB Orientifolds.” The benefits of such a local approach in F-theory have been advocated in particular in and following [124-126, 128].

截至目前的讨论都聚焦在规范代数的非阿贝尔部分、其带荷物质以及汤川型耦合。在一定程度上，通过分析沿一堆 7 膜的规范理论，加上其他膜与该堆膜相交曲线上的额外物质，就可以局域描述这些数据。局域方法相当于放大到判别式其中一个分支 $\sum_i$ 的邻域，这和“IIB 型定向模”小节中微扰框架下描述的局域方法类似。这种 F-theory 中局域方法的优势已在文献 [124-126, 128] 及后续工作中得到重点提倡。

Understanding non-Cartan abelian gauge symmetries, on the other hand, requires going beyond a local analysis of a given brane stack. As in the Type II orientifolds reviewed in section ”Type II Orientifolds,” non-Cartan abelian gauge symmetries depend on global, rather than local, data: the diagonal abelian gauge symmetries associated with the $U(N)$ gauge groups can acquire a Stückelberg mass, and only certain linear combinations of $U(1)$ s from different brane stacks remain as massless $U(1)$ s. Similarly, the existence of a massless non-Cartan $U(1)$ depends on global geometric properties of the elliptic fibration in F-theory.

另一方面，要理解非卡坦阿贝尔规范对称性，就不能局限于对给定膜堆的局域分析。正如“II 型定向共形场论”一节回顾的 II 型定向共形场论那样，非卡坦阿贝尔规范对称性依赖于整体数据而非局域数据: 与 $U(N)$ 规范群关联的对角阿贝尔规范对称性会获得施蒂克尔贝格质量，只有来自不同膜堆的 $U(1)$ 的特定线性组合才能保持为无质量 $U(1)$ 。类似地，无质量非卡坦 $U(1)$ 是否存在，取决于 F-理论中椭圆纤维化的整体几何性质。

The underlying construction builds on the definition of F-theory via duality with 11-dimensional M-theory. Compactification of M-theory on the same elliptic fourfold Y_4 gives a theory in three dimensions, which is identified with the compactification of F-theory on Y_4 times an additional circle S^1_{M} (see, e.g., the reviews [106,110] for details and references). In particular, this approach admits a detailed derivation of the F-theory effective action, as explained in [129]. To treat the M-theory compactification on Y_4 in supergravity, one must resolve the singularities in the elliptic fiber of Y_4 over Δ . Let us denote the fourfold after the resolution as $\hat{Y}_4$. At the abelian level, the gauge potentials in M-theory arise from the expansion of the M-theory three-form in terms of harmonic 2-forms of $\hat{Y}_4$:

> 上述基础构造建立在通过与 11 维 M 理论对偶定义 F-理论的框架之上。M 理论在同一个四维椭圆纤维化 Y_4 上紧致化后得到三维理论，该理论等价于 F-理论在 Y_4 乘以额外圆 S^1_{M} 上的紧致化 (例如详见综述 [106,110] 及其中参考文献)。具体而言，如文献 [129] 所述，该方法可以细致推导 F-理论的有效作用量。为了用超引力描述 M 理论在 Y_4 上的紧致化，必须解决 Y_4 在 Δ 上方椭圆纤维的奇点问题。我们将解决奇点后的四维流形记为 $\hat{Y}_4$ 。在阿贝尔层面，M 理论的规范势源自 M 理论三形式按 $\hat{Y}_4$ 调和二形式的展开:

$$C_3 = \sum_{\alpha=1}^{h^{1,1}(\hat{Y}_4)} A_\alpha \wedge \omega_\alpha + \ldots \tag{69}$$

for ω_α a basis of $H^{1,1}\left(\hat{Y}_4\right)$. Not all of these three-dimensional gauge potentials correspond to gauge fields in the four-dimensional F-theory. It turns out that only two types of 2-forms give rise to a gauge field in three dimensions which is associated with a 7-brane gauge field in F-theory.

> 其中 ω_α 是 $H^{1,1}\left(\hat{Y}_4\right)$ 的一组基。并非所有这些三维规范势都对应四维 F-理论中的规范场。实际上，只有两类二形式能在三维给出对应 F-理论中 7 膜规范场的规范场。

First, the resolution process from Y_4 to $\hat{Y}_4$ induces a set of exceptional divisors E_{α_i} on $\hat{Y}_4$ which are fibered by rational curves over the four-cycles $\sum_i$. Here $\alpha_i = 1, \ldots, \mathrm{rk}\,(G_i)$ runs over the generators of the Cartan sub-algebra of the Lie algebra G_i . The associated gauge potentials are the abelian Cartan gauge fields associated with the non-abelian gauge sector [130].

> 首先，从 Y_4 到 $\hat{Y}_4$ 的奇点解消过程会在 $\hat{Y}_4$ 上诱导出一组例外除子 E_{α_i} ，这些除子是有理曲线纤维化在四维循环 $\sum_i$ 上得到的。这里 $\alpha_i = 1, \ldots, \mathrm{rk}\,(G_i)$ 遍历李代数 G_i 卡坦子代数的生成元。关联的规范势就是与非阿贝尔规范 sector 关联的阿贝尔卡坦规范场 [130]。

Non-Cartan U(1) gauge potentials on the other hand require a different source of harmonic 2-forms: these are provided by so-called rational sections of $\hat{Y}_4$[114, 130 , 131]. A rational section embeds the base B_3 into $\hat{Y}_4$ as a divisor, and the different ways of doing so give additional independent dual harmonic 2-form classes. The set of rational sections S_a forms a finitely generated abelian group, the Mordell-Weil group $\mathrm{MW}\left(\hat{Y}_4\right)$, whose free part is in one-to-one correspondence with non-Cartan $U(1)_a$ gauge fields in F-theory. By taking a certain linear combination of the two-forms associated with the extra section, the zero-section, and other 2-form classes pulled back from B_3 , one obtains an element $\sigma\,(S_a) \in H^{1,1}\left(\hat{Y}_4\right)$ (called the image of the Shioda map) with the property that

另一方面，非卡坦 U(1) 规范势需要不同的调和二来源: 这类二形式由 $\hat{Y}_4$[114, 130 的所谓有理截面提供，[131]。一个有理截面将底空间 B_3 作为除子嵌入 $\hat{Y}_4$，不同的嵌入方式会给出额外独立的对偶调和二形式类。所有有理截面 S_a 构成一个有限生成阿贝尔群，即莫德尔-韦伊群 $\mathrm{MW}\left(\hat{Y}_4\right)$，其自由部分与 F-理论中的非卡坦 $U(1)_a$ 规范场一一对应。通过对额外截面、零截面以及从 B_3 拉回的其他二形式类关联的二形式做特定线性组合，可以得到满足如下性质的元素 $\sigma(S_a) \in H^{1,1}\left(\hat{Y}_4\right)$ (称为志田映射的像):

$$C_3 = \sum_{a=1}^{\mathrm{rk}\left(\mathrm{MW}\left(\hat{Y}_4\right)\right)} A_a \wedge \sigma(S_a) + \ldots \tag{70}$$

yields the non-Cartan gauge potentials A_a [132-134]. More details and a guide to the vast literature on this topic are provided in the reviews [110,111].

就得到非卡坦规范势 A_a [132-134]。更多细节以及该方向海量文献的导读可见综述 [110,111]。

A subtle point concerns that fate of the diagonal $U(1)$ gauge groups which play such an important role for model building in Type II orientifolds as detailed in sections "D-Brane Model Building: Generalities" and "Type II Orientifolds." For instance, in Type IIB orientifolds, the diagonal $U(1)$ gauge field in the $U(N)$ group can acquire a Stückelberg mass even before the effect of gauge fluxes is taken into account. We will refer to such $U(1)$ s as geometrically massive. Depending on the geometric details, the $U(1)$ gauge group is broken to a $\mathbb{Z}_k$ gauge group [38]. If $k \geq 2$, this manifests itself in a corresponding selection rule on the allowed couplings, while for $k = 1$, no such selection rules survive and the geometrically massive $U(1)$ cannot be detected at the level of the effective action. In F-theory, a geometrically massive $U(1)$ is realized directly in terms of its remnant discrete $\mathbb{Z}_k$ subgroup. Such discrete symmetries are associated with multi-sections which occur on genus-one fibrations not possessing a zero-section [115]. On the other hand, if a geometrically massive $U(1)$ does not leave behind a $\mathbb{Z}_k$ symmetry for $k \geq 2$, it only manifests itself at worst in certain $\mathbb{Q}$-factorial terminal singularities over curves in the base [135, 136].

一个微妙的问题关乎对角 $U(1)$ 规范群的命运，正如在“D 膜模型构建: 概述”和“II 型定向模”两节中所述，这类规范群在 II 型定向模的模型构建中发挥着至关重要的作用。例如，在 IIB 型定向模中，$U(N)$ 群内的对角 $U(1)$ 规范场甚至在考虑规范通量效应之前就可以获得施蒂克尔贝格质量。我们将这类 $U(1)$ 称为几何有质量的。根据几何细节的不同，$U(1)$ 规范群会破缺为 $\mathbb{Z}_k$ 规范群 [38]。若 $k \geq 2$，这会体现为对允许耦合的对应选择定则；而当 $k = 1$ 时，这类选择定则不复存在，几何有质量的 $U(1)$ 无法在有效作用量层面被探测到。在 F 理论中，几何有质量的 $U(1)$ 直接通过其残余离散 $\mathbb{Z}_k$ 子群实现。这类离散对称性与多截面相关，多截面出现在不具有零截面的亏格一纤维化上 [115]。另一方面，若几何有质量的 $U(1)$ 对 $k \geq 2$ 未留下 $\mathbb{Z}_k$ 对称性，它最多只会体现在底空间曲线上的某些 $\mathbb{Q}$-因子端点奇点中 [135, 136]。

Finally, the global structure of the 7-brane gauge group, as opposed to the gauge algebra, is encoded in the torsional part of the Mordell-Weil group [131, 137] in F-theory, with additional subtleties appearing in the presence of abelian gauge algebra factors [111,138] (As throughout this article, we will mostly not distinguish between the gauge group and the algebra in the sequel.).

最后，区别于规范代数，7 膜规范群的整体结构在 F 理论中编码在莫德尔-韦伊群的挠部分中 [131, 137]，当存在阿贝尔规范代数因子时还会出现额外的微妙之处 [111,138](同本文通篇一样，在下文中我们大多不区分规范群和规范代数。)

Apart from the gauge group supported on 7-branes, four-dimensional F-theory compactifications exhibit a gauge sector on spacetime-filling D3-branes as well as an abelian gauge sector in the Type IIB Ramond-Ramond sector [129, 139]. These extra gauge sectors cannot support chiral charged matter.

除了 7 膜承载的规范群外，四维 F 理论紧化还在充满时空的 D3 膜上存在规范扇区，且在 IIB 型的拉姆齐-拉姆齐扇区存在阿贝尔规范扇区 [129, 139]。这些额外的规范扇区无法支持手性带电物质。

Gauge Backgrounds: Stückelberg Terms and Matter Multiplicities

规范背景:Stückelberg 项与物质多重度

In addition to this purely geometric structure, the gauge background affects both the gauge algebra and the matter spectrum. This is in complete analogy to the effect of gauge background in Type IIB orientifolds; see section ”Type IIB Orientifolds.” For simplicity we will only consider abelian gauge backgrounds. Part of the information is encoded in the background value of the gauge field strengths along the compactified dimensions. In the language of M-theory, this corresponds to a background for the M-theory field strength $G_4 = dC_3$:

除这种纯几何结构外，规范背景会同时影响规范代数与物质谱。这和 IIB 型 orientifold 中规范背景的效应完全类似，参见“IIB 型 Orientifold”一节。为简化讨论，我们仅考虑阿贝尔规范背景。部分信息编码在紧致化维度上的规范场强背景取值中。用 M 理论的语言来说，这对应 M 理论场强 $G_4 = dC_3$ 的背景:

$$G_4 = \sum_{\alpha_i=1}^{\mathrm{rk}(G_i)} F_{\alpha_i} \wedge \left[E_{\alpha_i}\right] + \sum_{a=1}^{\mathrm{rk}\left(\mathrm{MW}\left(\hat{Y}_4\right)\right)} F_a \wedge [\sigma(S_a)] . \tag{71}$$

Here F_{α_i} and F_a take values in $H^{1,1}(B_3)$ and parametrize the internal part of the Cartan and non-Cartan $U(1)$ field strengths, respectively. The flux must satisfy the following constraints:

此处 F_{α_i} 和 F_a 取值于 $H^{1,1}(B_3)$，分别参数化 Cartan 与非 Cartan $U(1)$ 场强的内部分量。通量必须满足下列约束:

- The flux background is subject to a D3-brane tadpole equation, which can be elegantly written as [140]

- 通量背景满足 D3 膜蝌蚪方程，可以优雅地写为 [140]

$$n_{\mathrm{D3}} + \frac{1}{2}\int_{\hat{Y}_4} G_4 \wedge G_4 = \chi\left(\hat{Y}_4\right). \tag{72}$$

The Euler characteristic of $\hat{Y}_4$, $\chi\left(\hat{Y}_4\right)$, accounts for curvature contributions to the D3-brane tadpole on the 7-branes, and n_{D3} is the number of spacetime-filling D3-branes.

$\hat{Y}_4, \chi\left(\hat{Y}_4\right)$ 的欧拉示性数描述了 7 膜上对 D3 膜蝌蚪的曲率贡献，而 n_{D3} 是充满时空的 D3 膜的数目。

- The flux must obey the Freed-Witten quantization condition $G_4 + \frac{1}{2}c_2\left(\hat{Y}_4\right) \in H^2\left(\hat{Y}_4, \mathbb{Z}\right)$, where c_2 represents the second Chern class [141-143]. Whenever $\frac{1}{2}c_2\left(\hat{Y}_4\right)$ is not by itself an integer class, this condition implies that the gauge background must be nonvanishing in a consistent compactification.

- 通量必须满足 Freed-Witten 量子化条件 $G_4 + \frac{1}{2}c_2\left(\hat{Y}_4\right) \in H^2\left(\hat{Y}_4, \mathbb{Z}\right)$，其中 c_2 代表第二陈类 [141-143]。当 $\frac{1}{2}c_2\left(\hat{Y}_4\right)$ 本身不是整数类时，该条件意味着在自洽的紧致化中规范背景必须非零。

- The flux induces a D-term supersymmetry condition involving the Kähler moduli, which is satisfied if

- 通量诱导出包含 Kähler 模的 D 项超对称条件，该条件成立当且仅当

$$\int_{B_3} J \wedge F_A \wedge \pi_*\left(\omega_A \wedge \omega_B\right) = 0,\ \omega_A \in \left\{\left[E_{\alpha_i}\right], \left[\sigma\left(S_a\right)\right]\right\}. \tag{73}$$

This is essentially the condition (24). The F-term condition (23) translates into the condition $G_4 \in H^{2,2}_{\mathrm{vert}}$, the primary vertical subspace [144], which is automatically fulfilled for the choice (71).

这本质上就是条件 (24)。F 项条件 (23) 转化为主竖直子空间 [144] 上的条件 $G_4 \in H^{2,2}_{\mathrm{vert}}$，对于选择 (71) 该条件自动满足。

The gauge background must also be specified at the level of the underlying three-form potential in M-theory rather than merely the field strength. The gauge background is fully specified by an element of the Déligne cohomology group [145], which can partly be parametrized by the Chow group of $\hat{Y}_4$ [146,147].

规范背景也需要在 M 理论中底层三形式势层面而非仅场强层面指定。一个 Déligne 上同调群的元素可以完全确定规范背景 [145]，其中部分可以用 $\hat{Y}_4$ 的周环参数化 [146,147]。

The gauge background plays at least three different roles in the model building context in F-theory:

在 F 理论模型构建的语境下，规范背景至少发挥三种不同作用：

- A Cartan gauge background breaks the non-abelian gauge algebra to a subgroup involving abelian gauge factors. This effect will be discussed in more detail in the next section.

- Cartan 规范背景将非阿贝尔规范代数破缺到包含阿贝尔规范因子的子群。我们会在下一节详细讨论这一效应。

- Cartan and non-Cartan gauge fluxes both in general induce Stückelberg mass terms for the abelian gauge symmetries (along with a D-term potential for the Kähler moduli). If one collectively denotes by ω_A the two-forms $\left[E_{\alpha_i}\right]$ and $\sigma\left(S_a\right)$, then the Stückelberg mass matrix M_{AB} for the $U(1)_A$ gauge potentials is proportional to

- 一般而言，Cartan 与非 Cartan 规范通量都会为阿贝尔规范对称性诱导出 Stückelberg 质量项 (同时伴随 Kähler 模的 D 项势)。若我们用 ω_A 统一标记二形式 $[E_{\alpha_i}]$ 和 $\sigma(S_a)$，则对应 $U(1)_A$ 规范势的 Stückelberg 质量矩阵 M_{AB} 正比于

$$M_{AB} \propto \int_{\hat{Y}_4} G_4 \wedge \omega_A \wedge \omega_B. \tag{74}$$

For Standard Model constructions, it must therefore be checked whether the hypercharge $U(1)_Y$ remains massless in the presence of gauge backgrounds.

因此对标准模型构造而言，必须检验在存在规范背景的情况下，超荷 $U(1)_Y$ 是否仍保持无质量。

- The gauge background determines the multiplicities of the massless chiral superfields charged under the gauge algebra. In particular, the chiral index can be expressed as an integral of the form [128, 148-151]:

- 规范背景决定了规范代数下带荷无质量手征超场的多重度。具体来说，手征指标可以写为如下形式的积分 [128, 148-151]:

$$\chi(\mathbf{R}) = n_{\mathbf{R}} - n_{\overline{\mathbf{R}}} = \int_{S_{\mathbf{R}}} G_4, \tag{75}$$

where $S_{\mathbf{R}}$ is a complex surface on $\hat{Y}_4$ which can be attributed to every representation of the gauge group as detailed in [110] and references therein. This is the F-theory version of the expression (29). The amount of vector-like matter, i.e., the multiplicities $n_{\mathbf{R}}$ and $n_{\overline{\mathbf{R}}}$ rather than merely their difference, is likewise encoded in the gauge background, but is sensitive to the finer information contained in the Déligne cohomology group [146,147].

其中 $S_{\mathbf{R}}$ 是 $\hat{Y}_4$ 上的复曲面，如文献 [110] 及其中引用所述，该复曲面可对应规范群的每个表示。这就是表达式 (29) 的 F 论版本。类矢量物质的总量，即多重度 $n_{\mathbf{R}}$ 和 $n_{\overline{\mathbf{R}}}$，而非仅二者之差，同样也编码在规范背景中，且对德利涅上同调群包含的更精细信息敏感 [146,147]。

Standard Model Constructions

标准模型构造

There are two different classes of Standard Model realizations in F-theory.

F-理论中存在两类不同的标准模型实现方案。

Direct Standard Model Constructions

直接标准模型构造

The first approach is to directly engineer, in the geometry of the elliptic fourfold, Standard Model quivers of a form similar to the ones reviewed in section "D-Brane Model Building: Generalities." In such constructions, the gauge background controls the multiplicities of the massless charged matter fields and provides a mass term for additional abelian gauge fields other than hypercharge, if present. According to our discussion of the previous section, a geometric realization of a gauge algebra of the form

> 第一种方法是在椭圆四-fold 几何中直接构造出形式与“D 膜模型构建: 概述”章节所回顾的结构类似的标准模型箭图。在这类构造中，规范背景控制着零质量带电物质场的多重数，若除超荷外还存在其他额外阿贝尔规范场，规范背景会为其给出质量项。根据我们上一节的讨论，如下形式规范代数的几何实现

$$G = SU(3) \times SU(2) \times \prod_a U(1)_a \tag{76}$$

requires:

> 要求满足:

- a fibral singularity of either Kodaira Type I_3 or Type IV without monodromy along a divisor $\sum_{SU(3)}$,

> - 除因子 $\sum_{SU(3)}$ 所在除子外，纤维奇点为 Kodaira 类型 I_3 或无单模的 IV 型,

- a fibral singularity of Kodaira Type I_2 or Type III, or of Type IV with monodromy, along a divisor $\sum_{SU(2)}$,

> - 因子 $\sum_{SU(2)}$ 所在除子上，纤维奇点为 Kodaira 类型 I_2 、III 型或带单模的 IV 型,

- additional $U(1)_a$ gauge group factors as a consequence of extra rational sections of the fibration.

> - 额外的 $U(1)_a$ 规范群因子来自纤维化的额外有理截面。

Note that in direct Standard Model constructions which cannot be unhiggsed to a theory with a GUT group, $\sum_{SU(3)}$ and $\sum_{SU(2)}$ lie in different B_3 homology classes.

> 注意，在无法退希格斯化为大统一群理论的直接标准模型构造中，$\sum_{SU(3)}$ 和 $\sum_{SU(2)}$ 属于不同的 B_3 同调类。

One linear combination of the abelian group factors must correspond to hypercharge $U(1)_Y$, while the orthogonal linear combinations must acquire a Stückelberg mass by a suitable choice of gauge background. This approach to Standard Model building is comparable in spirit to the constructions in perturbative Type II orientifolds even though the F-theory framework is considerably more general and includes brane configurations which are not realizable in perturbative constructions. In particular, the engineering of the $SU(3)$ and $SU(2)$ factors via Kodaira fibers of Type IV or Type III has no perturbative analogue as it involves mutually nonlocal 7-brane stacks.

阿贝尔群因子的一个线性组合必须对应超荷 $U(1)_Y$ ，而正交的线性组合必须通过合适选取规范背景获得 Stückelberg 质量。尽管 F-理论框架普适性更强，包含了微扰构造无法实现的膜构型，这种标准模型构建方法的核心思想与微扰 II 型定向模构造是可比的。特别地，通过 IV 型或 III 型 Kodaira 纤维构造 $SU(3)$ 和 $SU(2)$ 因子不存在微扰对应，因为这涉及互相非局域的 7 膜堆。

The first realization of this approach in [152, 153] specializes a fibration with Mordell-Weil group of rank two constructed in [154, 155] to accommodate $SU(3) \times SU(2) \times U(1)_a \times U(1)_b$ (with the non-abelian part realized via I_3 and I_2 fibers), where in realistic constructions one linear combination of the abelian factors must be massive through the gauge background. Instead of an additional massive $U(1)$, dimension-four proton decay is prohibited in the construction of [156] by a discrete symmetry group $\mathbb{Z}_2$ (matter parity). In [157], one of the toric elliptic fibers dubbed F_{11} in [158] automatically encodes $(SU(3) \times SU(2) \times U(1))/\mathbb{Z}_6$ without additional symmetries to protect proton decay; this construction admits a plethora of chiral three-general models [159, 160]. A systematic analysis of $(SU(3) \times SU(2) \times U(1))/\mathbb{Z}_6$ models (containing these as special subcases) has been undertaken in [161-163]. Non-perturbative Standard Model with Kodaira fibers of Types IV and III are investigated in [164]; in such scenarios, for instance the SU(3) group can be non-higgsable [165,166], while the incorporation of the abelian factors is to date less well understood.

该方法的首个实现由文献 [152,153] 给出，他们将文献 [154,155] 构造的 Mordell-Weil 群秩为 2 的纤维化特殊化，以容纳 $SU(3) \times SU(2) \times U(1)_a \times U(1)_b$ (其中非阿贝尔部分通过 I_3 和 I_2 纤维实现)，在现实构造中，阿贝尔因子的一个线性组合必须通过规范背景获得质量。文献 [156] 的构造中没有额外的大质量 $U(1)$ ，四维质子衰变被离散对称群 $\mathbb{Z}_2$ (物质宇称) 禁戒。文献 [157] 中，文献 [158] 命名为 F_{11} 的托里克椭圆纤维自动编码了 $(SU(3) \times SU(2) \times U(1))/\mathbb{Z}_6$ ，不需要额外对称性来保护质子衰变；该构造容许大量手征三代模型 [159,160]。文献 [161-163] 对 $(SU(3) \times SU(2) \times U(1))/\mathbb{Z}_6$ 模型 (将上述构造作为特殊子情况包含在内) 进行了系统分析。文献 [164] 研究了带有 IV 型和 III 型 Kodaira 纤维的非微扰标准模型；在这类情景中，例如 SU(3) 群可以是不可退希格斯化的 [165,166]，而阿贝尔因子的引入目前还研究得不够充分。

GUT Constructions: Georgi-Glashow SU (5) GUTs

GUT 构造:Georgi-Glashow SU(5) GUT

The second approach is via Grand Unified Theories (GUTs) and was initiated in [124-126, 128]. This approach makes full use of the non-perturbative nature of the F-theory construction because unlike perturbative Type II orientifolds [72, 167], F-theory models admit a natural engineering also of the top quark Yukawa couplings with order one coefficients [124-128]. The general idea is to geometrically engineer a GUT group G containing $SU(3) \times SU(2) \times U(1)_Y$ and to break G to the latter by a sui choice of gauge background.

第二种方法通过大统一理论 (GUT) 实现，该方法始于文献 [124-126, 128]。这种方法充分利用了 F 理论构造的非微扰性质: 与微扰 II 型 orientifold [72, 167] 不同，F 理论模型也能自然地构造出系数为一阶量级的顶夸克汤川耦合 [124-128]。整体思路是通过几何方法构造出包含 $SU(3) \times SU(2) \times U(1)_Y$ 的 GUT 群 G ，再通过选择合适的规范背景将 G 破缺为 $SU(3) \times SU(2) \times U(1)_Y$ 。

In the context of F-theory, GUT groups $G = SU(5)$ (beginning with [124-126, 128]), $SO(10)$ [168] and E_6 [169-171] have been studied in detail (see also the reviews [107, 108] for further references). In the sequel we will illustrate the key ideas in the context of Georgi-Glashow $SU(5)$ GUT theories.

在 F 理论的框架下，GUT 群 $G = SU(5)$ (始于文献 [124-126, 128])、$SO(10)$ [168] 与 E_6 [169-171] 已经得到了细致研究 (更多参考文献也可参见综述 [107, 108])。下文我们将以 Georgi-Glashow $SU(5)$ GUT 理论为例阐释核心思想。

In Georgi-Glashow $SU(5)$ GUTs, the Standard Model gauge algebra is embedded into $SU(5)$:

在 Georgi-Glashow $SU(5)$ GUT 中，标准模型规范代数嵌入到 $SU(5)$ 中:

$$SU(3) \times SU(2) \times U(1)_Y \subset SU(5)\,. \tag{77}$$

In an $\mathcal{N} = 1$ supersymmetric framework, the charged matter content of the MSSM organizes into three generations of chiral multiplets transforming as the $\bar{\mathbf{5}}_{\mathbf{m}}$ and 10 representations of $SU(5)$:

在 $\mathcal{N} = 1$ 超对称框架中，最小超对称标准模型 (MSSM) 的带电物质内容组织为三代手征多重态，分别按 $SU(5)$ 的 $\bar{\mathbf{5}}_{\mathbf{m}}$ 和 10 表示变换:

$$\bar{\mathbf{5}}_{\mathbf{m}} \to (d^c, L) \tag{78}$$

$$\mathbf{10} \to (Q, u^c, e^+)\ . \tag{79}$$

The MSSM Higgs doublet is part of an additional vector-like pair of chiral multiplets, H_u and H_d , which arise from the decomposition:

MSSM 的希格斯二重态属于额外的矢量型手征多重态对 H_u 和 H_d ， 它们来自以下分解:

$$\mathbf{5}_H \to (T_u, H_u)\,,\ \bar{\mathbf{5}}_H \to (T_d, H_d)\,. \tag{80}$$

The triplets T_u and T_d do not exist in the MSSM and must be sufficiently heavy by the process of doublet-triplet splitting such that they are not only unobservable at the massless level but also do not induce dangerous dimension-five proton decay operators in the low-energy effective theory. Finally, extensions of the MSSM by right-handed neutrinos υ^c contain extra $SU(5)$ singlets.

三重态 T_u 和 T_d 不存在于 MSSM 中，必须通过双三重态分裂过程获得足够大的质量，这样它们不仅在无质量能级不可观测，也不会在低能有效理论中诱导危险的五维质子衰变算符。最后，MSSM 加入右手中微子 υ^c 的扩充模型包含额外的 $SU(5)$ 单态。

The MSSM Yukawa couplings are inherited from the two possible Yukawa couplings of the $SU(5)$ theory:

MSSM 的汤川耦合继承自 $SU(5)$ 理论的两种可能汤川耦合:

$$\mathbf{10105}_H \to Qu^cH_u \tag{81}$$

$$\mathbf{10}\bar{\mathbf{5}}_{\mathbf{m}}\bar{\mathbf{5}}_H \rightarrow Q d^c H_d + e^+ L H_d. \tag{82}$$

By contrast, Yukawa couplings of the form $\mathbf{10}\bar{\mathbf{5}}_{\mathrm{m}}\bar{\mathbf{5}}_{\mathrm{m}}$ or $\mathbf{10}\bar{\mathbf{5}}_H\bar{\mathbf{5}}_H$ would lead to phenomenologically unacceptable dimension-four proton decay operators and must hence be suppressed by additional selection rules which distinguish between the $\bar{\mathbf{5}}_{\mathbf{m}}$ and the $\bar{\mathbf{5}}_H$ representation.

反之，形式为 $\mathbf{10}\bar{\mathbf{5}}_{\mathrm{m}}\bar{\mathbf{5}}_{\mathrm{m}}$ 或 $\mathbf{10}\bar{\mathbf{5}}_H\bar{\mathbf{5}}_H$ 的汤川耦合会导出唯象上不可接受的四维质子衰变算符，因此必须通过额外的选择定则压低，这些选择定则会区分 $\bar{\mathbf{5}}_{\mathbf{m}}$ 表示与 $\bar{\mathbf{5}}_H$ 表示。

This general framework can be embedded into F-theory as follows:

这一整体框架可以按如下方式嵌入 F 理论:

GUT Group, Matter, Yukawas

GUT 群、物质、汤川耦合

The GUT group $SU(5)$ can be realized on a stack of 7-branes encoded in a Kodaira Type I_5 singularity in the elliptic fiber over a divisor $\sum_{SU(5)}$ on the base B_3 . Alternatively, the $SU(5)$ GUT group can also by itself be embedded into a higher gauge group which is broken accordingly by a gauge background, as studied systematically in [172, 173].

GUT 群 $SU(5)$ 可以实现于一组 7 膜上，对应底空间 B_3 中除子 $\sum_{SU(5)}$ 上方的椭圆纤维里的 Kodaira I_5 型奇点。或者，如文献 [172, 173] 的系统研究，$SU(5)$ GUT 群本身也可以嵌入更大的规范群，再通过规范背景破缺到该群。

Additional non-Cartan $U(1)$ or discrete gauge symmetries may be realized via the geometric mechanisms described in the previous section. At the geometric level, such extra symmetries require a further tuning of the Weierstrass model. The extra $U(1)$ gauge symmetries, if present, would have to be massive by a flux induced Stückelberg mechanism.

额外的非嘉当 $U(1)$ 或离散规范对称性可以通过上一节介绍的几何机制实现。在几何层面，这类额外对称性要求对魏尔斯特拉斯模型做进一步调参。如果存在额外的 $U(1)$ 规范对称性，它们会通过通量诱导的施蒂克尔贝格机制获得质量。

For definiteness, we will focus in the sequel on constructions with a geometrically tuned $SU(5)$ symmetry associated with a Kodaira Type I_5 fiber. Such a singularity can be conveniently engineered by applying Tate's algorithm [121,174], as reviewed in the $SU(5)$ GUT context in [108]. The $SU(5)$ charged matter multiplets are localized on curves on $\sum_{SU(5)}$ where the GUT brane stack intersects other 7-branes. The two relevant representations, $\mathbf{R} = \mathbf{10}$ and $\mathbf{R} = \mathbf{5}$, arise on curves $C_{\mathbf{R}}$ where the Kodaira type of the fibers enhance as follows:

为明确起见，下文我们将聚焦于具有几何调谐 $SU(5)$ 对称性、关联 Kodaira I_5 型纤维的构造。这类奇点可以通过泰特算法方便地构造出来 [121, 174]，文献 [108] 已经在 $SU(5)$ GUT 背景下综述了该方法。携带 $SU(5)$ 荷的物质多重态局域在 GUT 膜组与其他 7 膜相交的 $\sum_{SU(5)}$ 曲面上。两个相关表示 $\mathbf{R} = \mathbf{10}$ 和 $\mathbf{R} = \mathbf{5}$ 出现在纤维 Kodaira 类型按如下方式增强的曲线 $C_{\mathbf{R}}$ 上:

$$I_5(A_4) \to I_6(A_5): \;\; C_5 \tag{83}$$

$$I_5(A_4) \to I_1^*(D_5): \;\; C_{\mathbf{10}}. \tag{84}$$

In the parantheses we display the symmetry groups associated with these enhanced Kodaira singularities (see Table 7). In a Weierstrass model with an I_5 singularity over a divisor, both types of higher enhancements occur generically without further tuning. From a Type IIB orientifold point of view, C_5 is the intersection of the GUT brane stack with another 7-brane away from an orientifold plane, while C_{10} represents the intersection of the GUT brane stack with its orientifold image on top of an O7-plane. In the absence of extra massive $U(1)$ or discrete gauge groups differentiating between $\mathbf{5_m}$ and $\mathbf{5}_H$, both representations generically reside on the same matter curve, while extra such symmetries lead to a corresponding splitting of the matter curves. See also the discussion of Class (iii) models in section ”The Four Classes of Models.”

括号内我们给出了这些增强 Kodaira 奇点对应的对称群 (见表 7)。在除子上方带有 I_5 奇点的魏尔斯特拉斯模型中，两类更高阶增强无需额外调参就会一般地出现。从 IIB 型定向模的角度看，C_5 是 GUT 膜组与远离定向 O 平面的另一张 7 膜的交点，而 C_{10} 是 GUT 膜组与它在 O7 平面上的定向模像的交点。如果不存在区分 $\mathbf{5_m}$ 和 $\mathbf{5}_H$ 的额外有质量 $U(1)$ 或离散规范群，两种表示一般会位于同一条物质曲线上，而这类额外对称会导致物质曲线发生相应的分裂。另见“四类模型”一节中对 (iii) 类模型的讨论。

At the intersection points of the matter curves, the overlap of the matter wavefunctions gives rise to the Yukawa couplings allowed by gauge symmetry. The intersection points are characterized by further singularity enhancements:

在物质曲线的交点处，物质波函数的交叠产生了规范对称性允许的汤川耦合。交点的特征是进一步的奇点增强:

$$(I_6, I_1^*) \to \mathrm{IV}(E_6): \;\; \mathbf{10\,10\,5} \tag{85}$$

$$(I_6, I_1^*) \to I_2^*(D_6): \;\; \mathbf{10\,\overline{5}\,\overline{5}} \tag{86}$$

The second type of enhancement corresponds to a symmetry group $SO(12)$, and the associated couplings are present also in perturbative Type II orientifolds; the enhancement to a symmetry group E_6 , on the other hand, cannot be achieved perturbatively. The existence of such couplings is a trademark of mutually non-local $[p, q]$ 7-branes in F-theory.

第二类增强对应对称群 $SO(12)$ ，这类耦合也存在于微扰 IIB 型定向模中；另一方面，增强到对称群 E_6 无法通过微扰方式实现，这类耦合的存在是 F-论中互非局域 $[p, q]$ 7 膜的标志性特征。

Note that without extra $U(1)$ or discrete gauge symmetries, the $\bar{\mathbf{5}}_{\mathbf{m}}$ and the $\mathbf{5}_H$ representations are also indistinguishable at the level of the Yukawa couplings. In particular, this means that the phenomenologically unacceptable coupling $\mathbf{10}'\bar{\mathbf{5}}_{\mathbf{m}}\bar{\mathbf{5}}_{\mathbf{m}}$ or $\mathbf{10}'\bar{\mathbf{5}}_H\bar{\mathbf{5}}_H$ cannot be avoided without such selection rules [175].

注意，没有额外 $U(1)$ 或离散规范对称性时，$\bar{\mathbf{5}}_{\mathbf{m}}$ 和 $\mathbf{5}_H$ 表示在汤川耦合层面也是不可区分的。这意味着，如果没有这类选择定则，就无法避免唯象上不可接受的耦合 $\mathbf{10}'\bar{\mathbf{5}}_{\mathbf{m}}\bar{\mathbf{5}}_{\mathbf{m}}$ 或 $\mathbf{10}'\bar{\mathbf{5}}_H\bar{\mathbf{5}}_H$ [175]。

GUT Breaking

GUT 破缺

There are two different ways to break the $SU(5)$ GUT group to the Standard Model gauge group: either via a dynamically generated vacuum expectation value for a GUT Higgs or by a topological gauge background. The first mechanism faces the challenge of explaining the origin of the Higgs potential required for the GUT breaking. The smallest representation in which the GUT Higgs field can occur is the 24 representation, and there are two candidates for such a GUT Higgs: if the divisor $\sum_{\mathrm{SU(5)}}$ is nonrigid inside B_3 , there arise $h^0\left(\sum_{\mathrm{SU(5)}}, K_{\sum_{\mathrm{SU(5)}}}\right)$ massless chiral multiplets in the 24, whose bosonic components represent geometric deformation moduli of the brane stack. The Higgsing of $SU(5) \to SU(3) \times SU(2) \times U(1)_Y$ amounts to a geometric deformation of the brane stack into an intersecting brane model. The end result can equivalently be interpreted as a direct Standard Model construction in which the $SU(3)$ and $SU(2)$ brane stacks lie in the same homology class. Alternatively, if $h^1\left(\sum_{\mathrm{SU(5)}}\right) > 0$, there exists a corresponding number of continuous Wilson line moduli, whose VEV can likewise break the $SU(5)$ gauge group.

将 $SU(5)$ 大统一群破缺到标准模型规范群有两种不同方式: 一种是通过大统一希格斯获得动力学生成的真空期望值，另一种是通过拓扑规范背景。第一种机制面临的挑战是，需要解释大统一破缺所需希格斯势的起源。大统一希格斯场所能出现的最小表示是 24 维表示，这种大统一希格斯有两个候选: 如果除子 $\sum_{\mathrm{SU(5)}}$ 在 B_3 内部是非刚性的，就会产生处于 24 表示的 $h^0\left(\sum_{\mathrm{SU(5)}}, K_{\sum_{\mathrm{SU(5)}}}\right)$ 个无质量手征多重态，其玻色分量对应膜堆的几何形变模。对 $SU(5) \to SU(3) \times SU(2) \times U(1)_Y$ 的希格斯化相当于将膜堆几何形变为交叉膜模型。最终结果等价于可以解释为直接构造的标准模型，其中 $SU(3)$ 和 $SU(2)$ 膜堆处于同调类中。另一方面，如果 $h^1\left(\sum_{\mathrm{SU(5)}}\right) > 0$，则存在对应数量的连续威尔逊线模，它们的真空期望值同样可以破缺 $SU(5)$ 规范群。

In view of the difficulty of accounting for the symmetry breaking potential dynamically, the second, topological mechanism of GUT breaking is particularly attractive. The $SU(5)$ group can be broken to the Standard Model gauge group by an internal gauge background for the hypercharge $U(1)_Y$, the abelian subgroup of $SU(5)$ associated with a generator $T_Y = \mathrm{diag}\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}, -\frac{1}{2}, -\frac{1}{2}\right)$. This results in the following breaking pattern:

考虑到动力学解释对称破缺势存在困难，第二种拓扑大统一破缺机制格外有吸引力。通过对应超荷 $U(1)_Y$ 的内规范背景，$SU(5)$ 群可以破缺到标准模型规范群，这里超荷是 $SU(5)$ 中与生成元 $T_Y = \mathrm{diag}\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}, -\frac{1}{2}, -\frac{1}{2}\right)$ 关联的阿贝尔子群。由此得到如下破缺模式:

$$SU(5) \to SU(3) \times SU(2) \times U(1)_Y$$

(87)

$$\mathbf{24} \to (\mathbf{8},\mathbf{1},0) \oplus (\mathbf{1},\mathbf{3},0) \oplus (\mathbf{1},\mathbf{1},0) \oplus \left(\mathbf{3},\mathbf{2},\frac{5}{6}\right) \oplus \left(\bar{\mathbf{3}},\mathbf{2},-\frac{5}{6}\right)$$

$$\mathbf{10} \to \left(\mathbf{3},\mathbf{2},\frac{1}{6}\right) \oplus \left(\bar{\mathbf{3}},\mathbf{1},-\frac{2}{3}\right) + (\mathbf{1},\mathbf{1},1)$$

$$\mathbf{5} \to \left(\mathbf{3},\mathbf{1},-\frac{1}{3}\right) \oplus \left(\mathbf{1},\mathbf{2},\frac{1}{2}\right)$$

The 24 here refers to matter fields propagating on the 7-brane stack along $\sum_{SU(5)}$, so-called bulk matter. If $\pi_1\left(\sum_{SU(5)}\right)$ is non-trivial, the $U(1)_Y$ gauge background can be taken to be flat, corresponding to a discrete Wilson line. This discrete version of the continuous GUT breaking via Wilson line moduli is studied in [176].

此处的 24 指沿 $\sum_{SU(5)}$ 在 7 膜堆上传播的物质场，即所谓的体物质场。如果 $\pi_1\left(\sum_{SU(5)}\right)$ 是非平凡的，则 $U(1)_Y$ 规范背景可以取为平坦的，对应离散威尔逊线。这种通过威尔逊线模实现连续大统一破缺的离散版本已在文献 [176] 中研究。

Another possibility which is available more generally is to consider a hypercharge gauge background [128,177] characterized by a non-trivial line bundle L_Y on $\sum_{SU(5)}$ with $c_1\left(L_Y\right) = \frac{1}{2\pi}F_Y \in H^2\left(\sum_{SU(5)}\right)$. The hypercharge flux is subject to a number of conditions in order for the gauge-breaking mechanism to meet some basic phenomenological criteria:

另一种更具普适性的可能是考虑超荷规范背景 [128,177]，其特征是在 $\sum_{SU(5)}$ 上满足 $c_1\left(L_Y\right) = \frac{1}{2\pi}F_Y \in H^2\left(\sum_{SU(5)}\right)$ 的非平凡线丛 L_Y 。为了让规范破缺机制满足基本唯象学要求，超荷流量需要满足若干条件:

- A hypercharge gauge background induces a Stückelberg mass for the hypercharge gauge boson unless F_Y is cohomologically trivial on B_3 [178]. This condition can be written, at the level of cohomology, as

- 除非 F_Y 在 B_3 上是上同调平凡的 [178]，否则超荷规范背景会给超荷规范玻色子带来施蒂克尔贝格质量。该条件可以在上同调层面写为

$$\iota_! c_1\left(L_Y\right) = 0 \tag{88}$$

where $\iota : \sum_{\mathrm{SU(5)}} \to B_3$ denotes the embedding of the divisor $\sum_{\mathrm{SU(5)}}$ into the base and the Gysin map $\iota_! : H^2\left(\sum_{\mathrm{SU(5)}}\right) \to H^4\left(\sum_{\mathrm{SU(5)}}\right)$ is defined by first taking the Poincaré dual, then pushing forward at the level of homology and finally taking the Poincaré dual again. As follows from (74), the 4-form flux G_4^Y associated with such a line bundle must take value in the so-called remainder piece $H^{2,2}\left(\hat{Y}_4\right)_{\mathrm{rem}}$ [179] in the orthogonal decomposition:

其中 $\iota : \sum_{\mathrm{SU(5)}} \to B_3$ 表示除子 $\sum_{\mathrm{SU(5)}}$ 对底空间的嵌入，吉辛映射 $\iota_! : H^2\left(\sum_{\mathrm{SU(5)}}\right) \to H^4\left(\sum_{\mathrm{SU(5)}}\right)$ 的定义为: 先取庞加莱对偶，再在同调层面推前，最后再次取庞加莱对偶。由式 (74) 可得，与该线丛关联的 4-形式流 G_4^Y 必须取值于正交分解中所谓的剩余块 $H^{2,2}\left(\hat{Y}_4\right)_{\mathrm{rem}}$ [179]:

$$H^{2,2}\left(\hat{Y}_4\right) = H^{2,2}\left(\hat{Y}_4\right)_{\mathrm{hor}} \oplus H^{2,2}\left(\hat{Y}_4\right)_{\mathrm{vert}} \oplus H^{2,2}\left(\hat{Y}_4\right)_{\mathrm{rem}}, \tag{89}$$

where the first two summands refer to the primary horizontal and primary vertical subspaces of $H^{2,2}\left(\hat{Y}_4\right)$, respectively.

其中前两项分别对应 $H^{2,2}\left(\hat{Y}_4\right)$ 的原水平子空间和原竖直子空间。

- The second condition on the gauge background arises by demanding that there occur no exotic massless states in the representation $\left(\mathbf{3}, \mathbf{2}, \frac{5}{6}\right) + c.c$ from the decomposition (87) of the 24 representation. These so-called lepto-quarks are absent only if all cohomology groups of $L_Y^{\pm 5/6}$ on $\sum_{SU(5)}$ vanish:

- 规范背景的第二个条件要求，24 表示分解 (87) 得到的 $\left(\mathbf{3}, \mathbf{2}, \frac{5}{6}\right) + c.c$ 表示中不存在奇异零质量态。仅当 $L_Y^{\pm 5/6}$ 在 $\sum_{SU(5)}$ 上的所有上同调群都为零时，这类所谓的轻夸克才会消失:

$$H^i\left(L_Y^{\pm 5/6}\right) = (0, 0, 0) \tag{90}$$

which is a strong constraint. The power of $\frac{5}{6}$ reflects the $U(1)_Y$ charge of the lepto-quarks. It turns out [125, 128] that rather than an integral line bundle L_Y, one can consider a fractionally quantized line bundle $\mathcal{L}_Y$ with the property that the lepto-quarks are counted by the cohomology group $H^i\left(\mathcal{L}_Y^{\pm 1}\right), i = 0, 1, 2$. The vanishing of these cohomology groups can be achieved for instance by taking $\mathcal{L}_Y = E_i - E_j$ on a del Pezzo surface dP_n with exceptional curve classes E_i, $i = 1, \ldots n$.

这是一个很强的约束。$\frac{5}{6}$ 的幂次反映了轻夸克的 $U(1)_Y$ 电荷。结果表明 [125, 128]，相比于整号线丛 L_Y，我们可以考虑分数量子化线丛 $\mathcal{L}_Y$，其性质是轻夸克由上同调群 $H^i\left(\mathcal{L}_Y^{\pm 1}\right), i = 0, 1, 2$ 计数。例如，在带例外曲线类 E_i、$i = 1, \ldots n$ 的德尔佩佐曲面 dP_n 上取 $\mathcal{L}_Y = E_i - E_j$，就可以实现这些上同调群为零。

- The Standard Model matter should appear in complete GUT multiplets. This means that

- 标准模型物质必须以完整的大统一多重态出现。也就是说

$$\int_{C_{10}} F_Y = 0 = \int_{C_{\bar{5}\mathbf{m}}} F_Y \tag{91}$$

The requirement (91) is only a necessary condition which guarantees that the chiral index of the MSSM representations do not differ within a GUT multiplet. This must in fact be ensured also at the vector-like level.

要求 (91) 只是一个必要条件，它保证最小超对称标准模型表示的手征指标在一个大统一多重态内没有差异。实际上这一点在矢量型层面也必须得到保证。

Apart from being topological, one of the benefits of hypercharge GUT breaking is that the doublet-triplet splitting problem can be solved by a suitable restriction of the hypercharge gauge background to the Higgs curve. This will be discussed momentarily.

除拓扑性质外，超荷大统一破缺的一个优点是，通过将超荷规范背景适当地限制在希格斯曲线上，可以解决二重态-三重态劈裂问题。我们接下来就讨论这一点。

On the other hand, the hypercharge flux breaking affects precision gauge coupling unification via flux-dependent subleading corrections to the gauge kinetic function, as pointed out first in [128, 180]. This effect may be counterbalanced by the appearance vector-like exotics for instance at intermediate scales [181].

另一方面，正如文献 [128, 180] 最早指出的，超江流破缺会通过流依赖的规范动力学函数次领头修正影响精确规范耦合统一。例如，中间尺度出现的矢量型奇异粒子可以抵消这一效应 [181]。

Proton Decay and Selection Rules

质子衰变与选择规则

A closer analysis of this and related effects and more generally of the detailed realisation of the matter spectrum requires specifying the additional selection rules invoked to prevent the phenomenologically excluded proton decay operators by which GUT models are typically inflicted. As a minimal requirement, the extra symmetry must distinguish the $\mathbf{5}_{\mathbf{m}}$ and $\mathbf{5}_H$ representations and forbid the dangerous dimension-four operators $10\bar{\mathbf{5}}_{\mathbf{m}}\bar{\mathbf{5}}_{\mathbf{m}}$ and $10\bar{\mathbf{5}}_H\bar{\mathbf{5}}_H$ while allowing for the Yukawa couplings $\mathbf{10}_{\bar{5}_{\mathbf{m}}}\bar{\mathbf{5}}_H$ and $\mathbf{10105}_H$. Up to an overall normalization, and assuming for now that the charge assignments do not distinguish between the three matter families, these conditions are met by an extra (massive) $U(1)$ gauge symmetry with charge assignments:

要对该效应及相关效应进行更细致的分析，更一般地说，要详细实现物质谱，就必须明确额外的选择规则——这些规则用来排除 GUT 模型中普遍存在、被实验排除的质子衰变算符。作为最低要求，额外对称性必须区分 $\mathbf{5}_{\mathbf{m}}$ 和 $\mathbf{5}_H$ 表示，禁戒危险的四维算符 $10\bar{\mathbf{5}}_{\mathbf{m}}\bar{\mathbf{5}}_{\mathbf{m}}$ 和 $10\bar{\mathbf{5}}_H\bar{\mathbf{5}}_H$ ，同时允许汤川耦合 $\mathbf{10}_{\bar{5}_{\mathbf{m}}}\bar{\mathbf{5}}_H$ 和 $\mathbf{10105}_H$ 。在整体归一化下，且暂时假设电荷分配不区分三代物质家族，满足这些条件的是一个额外 (有质量) $U(1)$ 规范对称性，其电荷分配为:

$$\mathbf{10}_1,\ \left(\bar{\mathbf{5}}_{\mathbf{m}}\right)_{-q},\ \left(\mathbf{5}_H\right)_{-2},\ \left(\bar{\mathbf{5}}_H\right)_{q-1},\ q \neq \frac{1}{2}, \tag{92}$$

or a suitable discrete subgroup thereof.

或其合适的离散子群。

There are then two general possibilities: the assignment $q = 3$ corresponds to the so-called $U(1)_X$ symmetry, under which the charges of H_u and H_d merely differ by a sign. In a geometric realisation of such an additional symmetry (and no additional symmetries on top), H_u and H_d form a vector-like pair localized on a single curve $C_{\mathbf{5}_H}$, and altogether such models have three different types of localized matter curves, $C_{\mathbf{10}_1}, C_{(\mathbf{5}_{\mathbf{m}})_{-3}}$, and $C_{(\mathbf{5}_H)_{-2}}$ [175]. The first globally consistent realisations of this model with three chiral generations of MSSM matter have been constructed in [150].

接下来存在两种普遍情况: 分配 $q = 3$ 对应所谓的 $U(1)_X$ 对称性，在此对称性下 H_u 和 H_d 的电荷仅相差一个符号。在这种额外对称性的几何实现中 (且不存在更多额外对称性)，H_u 和 H_d 构成定域在单条曲线 $C_{\mathbf{5}_H}$ 上的矢量对，这类模型总共存在三种不同类型的定域物质曲线: $C_{\mathbf{10}_1}, C_{(\mathbf{5}_{\mathbf{m}})_{-3}}$ 和 $C_{(\mathbf{5}_H)_{-2}}$ [175]。该模型拥有三代 MSSM 手征物质的第一个整体一致实现在文献 [150] 中完成。

The $U(1)_X$ charge assignment has the disadvantage that it does not forbid dimension-five proton decay operators. To prevent these from being generated, the additional $U(1)$ (or, for that matter, discrete $\mathbb{Z}_k$) selection rule must distinguish also between H_u and H_d other than just by an overall sign of the charge, which fixes $q \neq 3$. A $U(1)$ gauge symmetry with this property is said to be of Peccei-Quinn (PQ) type [125, 128]. The different charge assignments have been studied intensively in the F-theory literature [181-185], to which we refer for details and further references.

$U(1)_X$ 电荷分配的缺点是无法禁戒五维质子衰变算符。为了阻止这类算符产生，额外的 $U(1)$ (或就此而言，离散 $\mathbb{Z}_k$) 选择规则除了区分整体电荷符号外，还必须区分 H_u 和 H_d ，这就固定了 $q \neq 3$ 。拥有该性质的 $U(1)$ 规范对称性被称为佩西-奎因 (PQ) 型 [125, 128]。不同的电荷分配已在 F 理论文献中得到深入研究 [181-185]，详情和更多参考文献可参见这些文献。

Note that the above discussion assumes that the extra selection rule does not distinguish between the three different families of matter within the same MSSM representation. Without this requirement a plethora of new symmetry patterns opens up. At a phenomenological level, distinguishing between families can explain the hierarchical structure of Yukawa couplings via the Froggatt-Nielsen mechanism [186, 187].

请注意，上述讨论假设额外选择规则不区分同一 MSSM 表示中的三代不同物质家族。若不满足该假设，就会出现大量新的对称性模式。从唯象角度看，区分不同代可以通过弗罗加特-尼尔森机制解释汤川耦合的层级结构 [186, 187]。

Gauge Background

规范背景

The hypercharge flux must be complemented by additional gauge background which controls the multiplicities of charged matter, in particular the chiral index (75). At the level of 4-form flux, the flux background takes the form:

超荷通量必须辅以额外的规范背景来控制带电物质的多重度，尤其是手征指数 (75)。在 4-形式通量层面，通量背景的形式为:

$$G_4 = G_4^{\mathrm{c}} + G_4^Y \tag{93}$$

where $G_4^Y \in H^{2,2}_{\mathrm{rem}}\left(\hat{Y}_4\right)$ denotes the hypercharge flux background and G_4^{c} is the part of the flux background which is blind to the GUT group breaking. To obtain the correct chiral index of MSSM matter, the flux must satisfy the following conditions:

其中 $G_4^Y \in H^{2,2}_{\mathrm{rem}}\left(\hat{Y}_4\right)$ 表示超荷通量背景，G_4^{c} 是通量背景中对 GUT 群破缺无影响的部分。要得到最小超对称标准模型物质正确的手征指数，通量必须满足以下条件:

$$\int_{S_{\mathbf{R}_{\vec{q}}}} G_4^Y = 0, \sum_{\vec{q}} \int_{S_{\mathbf{R}_{\vec{q}}}} G_4^{\mathrm{c}} = 3, \ \mathbf{R} = \mathbf{10}, \bar{\mathbf{5}}_{\mathbf{m}}. \tag{94}$$

Here $S_{\mathbf{R}_{\vec{a}}}$ is the matter surface associated with representation $\mathbf{R}_{\vec{q}}$ of $U(1)$ charge vector $\vec{q} = (q_1, \ldots, q_n)$, where we are allowing for n additional massive abelian gauge groups. The first condition is essentially (91) and ensures that all representations appear in complete GUT multiplets.

此处 $S_{\mathbf{R}_{\vec{a}}}$ 是对应表示 $\mathbf{R}_{\vec{q}}$ 、$U(1)$ 电荷向量 $\vec{q} = (q_1, \ldots, q_n)$ 的物质曲面，我们这里考虑了 n 个额外的有质量阿贝尔规范群。第一个条件本质上就是 (91)，它保证所有表示都以完整 GUT 多重态的形式出现。

As for the Higgs sector, let us specialize for definiteness to a single extra massive $U(1)$ gauge group with charges (92). If $q = 3$, one requires that at the chiral level

至于希格斯区，我们明确限定为单个带有电荷 (92) 的额外有质量 $U(1)$ 规范群。若满足 $q = 3$ ，则要求在手征层面满足:

$$\int_{S_{5_H}} G_4^Y = 0, \int_{S_{5_H}} G_4^c = 0, \tag{95}$$

but the gauge background must give rise to one vector-like pair of massless H_u and H_d fields while both triplets T_u and T_d must be absent at the massless level. This means that the restriction of G_4^c and G_4^Y must describe a line bundle of trivial curvature whose cohomology groups are compatible with this vector-like spectrum. If, on the other hand, the abelian symmetry is of PQ type $(q \neq 3)$, then doublet triplet splitting can already be imposed at the chiral level:

但规范背景需要产生一对无质量的矢量型 H_u 和 H_d 场，同时在无质量能谱中不存在三重态 T_u 和 T_d 。这意味着 G_4^c 和 G_4^Y 的限制必须描述一个曲率平凡的线丛，其上同调群与该矢量型能谱相容。另一方面，如果阿贝尔对称性是 PQ 型 $(q \neq 3)$ ，那么双态-三重态劈裂可以直接在手征层面要求:

$$\begin{aligned} \int_{S_{T_u}} G_4^Y + G_4^c = 0 = \int_{S_{T_d}} G_4^Y + G_4^c, \\ \int_{S_{H_u}} G_4^Y + G_4^c = 1 = \int_{S_{H_d}} G_4^Y + G_4^c. \end{aligned} \tag{96}$$

Here we have split the matter surface into a surface associated with the triplets and the doublets in the decomposition (80). Both these surfaces share the same base curve, but differ in the fiber. On top of these conditions, no vector-like pairs of states must be generated by the flux background to generate the exact massless MSSM matter content.

我们此处将物质曲面分解为对应分解 (80) 中三重态和双态的两个曲面，这两个曲面共享相同的基曲线，但纤维不同。除了这些条件，通量背景不能产生额外的矢量型态对，才能得到精确的无质量最小超对称标准模型物质内容。

The second condition of (94) controlling the chiral matter content was for the first time realized in globally consistent SU(5) GUTs in [150]. The conditions (95) have been exemplified in simple toy models in the literature [188, 189]. Their counterpart in PQ-type models (96), has not yet been realized, as of this writing, in globally consistent F-theory models where the hypercharge flux satisfies in addition the important constraint

(88). At the same time, considerations in the weak coupling limit suggest that this should be achievable in principle [188].

(94) 中控制手征物质内容的第二个条件首次在文献 [150] 的整体一致 SU(5)GUT 中实现。条件 (95) 已经在文献的简单玩具模型 [188, 189] 中给出示例。截至本文撰写，PQ 型模型中对应的条件 (96) 尚未在整体一致的 F 理论模型中实现——这类模型中的超荷通量还需要满足重要约束 (88)。同时，弱耦合极限下的研究表明，原则上这是可以实现的 [188]。

Yukawa Couplings and Flavor Hierarchies

汤川耦合与味层级

The Yukawa couplings can in principle be computed locally by evaluating the overlap of the matter wavefunctions at the intersection points of the matter curves. The local structure of the Yukawa couplings favors a mass hierarchy among the different families whose wavefunctions overlap at the same point. For details and a guide to the literature, we refer to [50, 190-195]. A remaining challenge for the future, however, is to connect such local computations to the data of the globally defined F-theory model. An alternative approach to explaining flavor hierarchies is via the Froggatt-Nielsen mechanism [186, 187].

汤川耦合原则上可以通过计算物质波函数在物质曲线交点处的重叠积分，进行局域计算。汤川耦合的局域结构有利于不同家族之间形成质量分层，这些家族的波函数在同一点发生重叠。相关细节和文献指引可参考 [50, 190-195]。但未来仍有一个待解决的挑战，就是将这类局域计算与全局定义的 F 理论模型的数据联系起来。解释味分层的另一种方法是通过 Froggatt-Nielsen 机制 [186, 187]。

Acknowledgments We thank I. García-Etxebarria, F. Quevedo, A. Sagnotti, and A. M. Uranga for discussions. FM is supported through the grants CEX2020-001007-S and PID2021-123017NB-I00, funded by MCIN/AEI/10.13039/501100011033 and by ERDF A way of making Europe. TW is supported in part by Deutsche Forschungsgemeinschaft under Germany’s Excellence Strategy EXC 2121 Quantum Universe 390833306 and by Deutsche Forschungsgemeinschaft through a German-Israeli Project Cooperation (DIP) grant ”Holography and the Swampland.”

致谢我们感谢 I. García-Etxebarria、F. Quevedo、A. Sagnotti 与 A. M. Uranga 的讨论。FM 得到了 CEX2020-001007-S 与 PID2021-123017NB-I00 项目资助，这些项目由 MCIN/AEI/10.13039/501100011033 以及欧洲区域发展基金“欧洲发展之路”资助。TW 部分得到德国研究基金会基于德国卓越战略 EXC 2121“量子宇宙”390833306 的资助，同时得到德国研究基金会德以项目合作 (DIP) 基金“全息与沼泽地”的支持。

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